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# Fuzzy Goal Programming Problem Based on Minmax Approach for Optimal System Design

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**ABSTRACT**

Every system in nature evolved in order to carry on their existence and reach their targets with minimal losses. The fundamental condition of a system's success lies on making the correct decision by evaluating multiple, complicated, and conflicting goals based on the present constraints. Many mathematical programming problems are makeup of objective functions combined by the decision maker based on the constrains. This study investigates how an optimal design can be reached based on Minmax approach. Goal Programming and a Fuzzy Goal Programming known as MA approach are used in this study. The solution of a problem organized as a Multiple De novo programming in order to determine the resource amounts for a business in handcrafts is carried out based on these two approaches. Budget constrain is organized as a goal to solve the problem based on MA approach, and a solution is proposed accordingly. The acquired results suggest that the solution results of Minmax Goal Programming and MA approach are the same.

**Keywords:** De Novo Programming, Fuzzy Goal Programming, Minmax Goal Programming, Optimal System Design

# Optimal Sistem Tasarımı İçin Minmaks Tabanlı Bulanık Hedef Programlama Kullanımı

**ÖZ**

Tabiattaki bütün sistemler, varlıklarını devam ettirmek ve hedeflerine en az kayıpla ulaşmak için zaman içerisinde değişim geçirmişlerdir. Sistemlerin başarıya ulaşabilmelerinin temel şartı birden fazla, ihtilaflı ve karmaşık amaçları mevcut kısıtlara göre değerlendirip en doğru kararı verebilmektir. Birçok matematiksel programlama problemi, karar verici tarafından kısıtlara bağlı olarak amaç fonksiyonlarının bir araya getirilmesinden oluşmaktadır. Bu çalışmada Minmaks tabanlı yaklaşımla optimal sistemin tasarımının nasıl yapılacağı araştırılmıştır. Araştırmada Minmaks Hedef Programlama ile MA yaklaşımı olarak da bilinen bir Bulanık Hedef yaklaşımı kullanılmıştır. El sanatları üretimi yapan bir işletmede kaynak miktarlarının optimal seviyede belirlenebilmesi için Çok Amaçlı De novo programlama olarak kurulan problemin çözümü bu iki yaklaşıma göre yapılmıştır. MA yaklaşımına göre problemin çözülebilmesi için bütçe kısıtı bir hedef olarak düzenlenmiş ve bir çözüm önerisi yapılmıştır. Elde edilen sonuçlara göre Minmaks Programming ve MA yaklaşımının çözüm sonuçlarının aynı olduğu belirlenmiştir.

**Anahtar Kelimeler:** De Novo Programlama, Bulanık Hedef Programlama, Minmaks Hedef Programlama, Optimal Sistem Tasarımı

## 1. Introduction

Mathematical models aim to reach a certain goal under given constrains.

When resource utilization is analysed based on model solution, it is mostly either surplus or shortage. Such an expected case hinders optimal production in terms of resources and causes decreases in profitability. The key for businesses is to quit the idea of maximum profit or minimum cost and to form the optimal production model by using their resources at full capacity. Therefore, constrain functions should be taken into account instead of objective functions when dealing with a business problem. It is because the goals are directed effected from the constrains.

While the constrains and constraint sources in traditional Linear Goal Programming are constant, De novo approach enables the redesign of resources thanks to the flexibility of constrain sources (Zeleny, 1984). An optimal system design states how an optimal system can be arranged based on maximum goal success values and full capacity use of constrains, instead of optimizing a given system. An optimal production plan can be possible with the optimal use of raw material amounts (Babic & Pavic, 1996). Additionally, an optimal system not only determines the best mixture of all outputs but also that of the inputs (Tabucanon, 1988). A system design, redesign, and optimization must include the reformation of system limits and constrains based on goals. System design is not a selection of alternatives but a creation of alternatives (Zeleny, 1986). This study uses to minmax based Goal Programming approaches to create an optimal design on a production process. First, the organization of a De Novo Programming model based on MA (Yaghoobi & Tamiz, 2007) which is a Fuzzy Goal Programming approach is demonstrated. Here, budget is regarded as a fuzzy goal and explained as a "triangular fuzzy number" based on the acceptable tolerance amount. The same problem is handled according to the Minmax Goal Programming developed by Flavell (1976), and the solution is carried out with budget as a goal along with the other goals.

## 2. De Novo Programming

Instead of optimizing a system, Zeleny (1976) conducted the first study on De Novo Programming proposing to design the optimal system. According to Zeleny (1984) De Novo Programming enables optimal design thanks to the long-term restructuring of resources, more efficient use of scarce resources, and prevention of wastefulness. While de novo hypothesis was applied only to classical linear programming problems in the beginning, it can easily be applied to multiobjective linear programming problems. Multi Criteria De Novo Programming problem proposed by Zeleny (1990) is given mathematically below. The formulation for maximization and minimization directed objectives are reorganized for generalization.

$$Max\ Z_k = C^k x_j$$
$$Min\ W_s = W^s x_j$$
Subject to (M1.1)
$$Ax - b \le 0$$
$$pb \le B$$
$$x_j \ge 0$$

where, $Z_k = C^k x_j = \sum_{j=1}^{n} C_{kj} x_j, k = 1,2,\dots,l$ are objective functions $Z_k$ to be maximized simultaneously. $W_s = C^s x_j = \sum_{j=1}^{n} C_{sj} x_j, s = 1,2,\dots,r$ objective functions $W_s$ to be minimized simultaneously. $C^k \in R^{lxn}, C^s \in R^{rxn}$ and $A \in R^{mxn}$ are matrices of dimensions $lxn, rxn$ and $mxn$ respectively.$b \in R^m$ is m-dimensional unknown resources vector, $p \in R^m$ is vector of unit prices of m resource vector, and B is the given total budget. (M1.1) can be rewritten as seen below based on budget constrain

$Max\ Z_k = C^k x_j$
$Min\ W_s = W^s x_j$
Subject to (M1.2)
$Vx \leq B$
$x_j \geq 0$
Here $V = (V_1, V_2, \dots, V_n) = pA \in R^n$.

Although there is no general specific solution method for Multicriteria De Novo Programming problems, Zeleny (1986) used the concept of "meta-optimal" for optimal system design. Afterwards, Zeleny (1990) explained the details of meta-optimality approach for the solution of (M1.2). Positive ideal solutions must be acquired for each objectives function for meta-optimality approach. Positive ideal solutions are acquired from the solution of each objective function based on their given direction. Positive ideal solutions are also named as the best performance of each goal function in (M1.1) or (M1.2).

$$I^* = \{Z_1^*, Z_2^*, \dots, Z_l^*; W_1^*, W_2^*, \dots, W_r^*\} \quad (1)$$

Negative ideal solutions are not used in meta-optimality approach. As the solution proposed in this study uses negative ideal solutions, they are explained as well. Negative ideal solutions are the solutions that minimize the maximization directed objectives and maximize the minimization directed objectives. Negative ideal solutions are named as the worst performance of each objective function in (M1.1) or (M1.2).

$$I^- = \{Z_1^-, Z_2^-, \dots, Z_l^-; W_1^-, W_2^-, \dots, W_r^-\} \quad (2)$$

Meta-optimal problem is formed as seen below.

$Min\ B = Vx$
Subject to; (M1.3)
$C^k x_j = Z_k^*$
$W^s x_j = W_s^*$
$Vx \leq B$
$x_j \geq 0$

With the solution of (M1.3), one can obtain $x^*$, $B^* = Vx^*$. $B^*$value is named as meta-optimum budget. Solving (M1.3) identifies the minimum budget $B^* = Vx^*$. at which the metaoptimum performance $Z_k^*$ and $W_s^*$ can be realized through $x^*$ and $b^*$. Solving (M1.3) must exceed any given budget B. Optimum-path ration "r" can be used with a pre-defined budget "B". $r = \frac{B}{B^*}$. Using "r", final solution formulations can be defined as: $x = rx^*$, $b = rb^*$, $Z_k = rZ_k^*$ and $W_s = rW_s^*$.

Shi (1995) proposed a new approach to solve De Novo Programming problems and defined six different types of optimum-path ratio. Furthermore, Min-Max Goal Programming, bound to positive ideal solutions and negative ideal solutions, are used by Umarusman (2013). Umarusman and Türkmen (2013) proposed the Global Criteria Method based on positive ideal solutions to solve Multi Criteria De Novo Programming problems. Zhuang and Hocine (2018) put forward Meta-Goal Programming in solutions of Multi Criteria De Novo Programming problems. Babic and Pavic (1996), Shi (1999), Chen and Hsieh (2006), Chakraborty and Bhattacharya (2013), Huang et al. (2016), Zhang et al. (2009), and Chen and Tzeng (2009) contributed to the Multiple Criteria De Novo Programming literature with their studies. When De Novo Programming problems are considered in terms of the process of "decision making in fuzzy environment", there are four main approaches. First, Lai and Hwang (1992) used Chanas' (1983) non-symmetric approach to solve single criterion de Novo Programming problem. Later, did Multi criteria De Novo Programming with fuzzy parameters based on the concept of fuzzy clustering probability. Li and Lee (1990) proposed a two-phase fuzzy approach based on positive and negative ideal solutions for Multi Criteria De Novo Programming. The fuzzy model in their study considers all parameters as fuzzy and defines membership functions for those parameters. In their next article which is considered to be a continuation of their first study where they analysed Multi Criteria De Novo Programming in a fuzzy logic frame, Lee and Li (1993) studied fuzzy goals and fuzzy coefficients simultaneously and proposed a different approach.

## 3. Fuzzy Goal Programming

Goal programming studies were originally started by Charnes et al. (1955). Later Charnes and Cooper (1961) formulated Goal Programming. Goal Programming aims to minimize deviation from aspired levels set by the decision maker and carries that minimization process with various methods. There are three fundamental Goal Programming methods: The first study on Archimedean Goal Programming was carried out by Ijiri (1965) who considered priority and weight factors together. Later Charnes and Cooper (1977) formulated Archimedean Goal Programming Model. Charnes and Cooper (1977) took out the weight factors from Ijiri's (1965) original study and proposed the model which only consisted of priority ranking for each goal. Minmax Goal Programming which was developed by Flavell (1976) minimizes maximum deviation instead of the sum of deviating variables, which is different from the weighted and prioritized structures of Goal Programming. After the establishment of "Fuzzy Sets" theory by Zadeh (1965), Bellman and Zedeh (1970) proposed the process of Decision Making in Fuzzy Environment, which helped the development of many approaches in Goal Programming methods as in Linear Programming.

The literature summary of scientific articles in the theorical development process of Fuzzy Goal Programming can be stated as follows. The primary studies in Fuzzy Goal Programming were initiated by Narasimhan (1980) who proposed a fuzzy goal programming model in which both the goals and the priorities are treated as fuzzy variables. Hannan (1981a) introduced interpolated membership functions (or piecewise linear membership functions) into the fuzzy goal programming model. Narasimhan (1981) used fuzzy weights for the first time in his proposed approach. Hannan (1981 a,b) proposed in his model $w_i^+$ and $w_i^-$ coefficients which stand for the relative significances for positive and negative deviations. Rubin and Narasimhan (1984) proposed a new approach to formulating fuzzy priorities in a goal programming problem. Tiwari, Dharma & Rao (1986) investigated how "preemptive priority" structure could be used in FGP and proposed an algorithm for solution. Tiwari et al. (1987) also have formulated the FGP problem with a simple additive model. Yang et al. (1991) proposed a model for solving FGP problems with triangular linear membership functions. Wang & Fu (1997) proposed a method to solve the FGP problem with preemptive priority structure via utilizing a penalty cost. Chen and Tsai (2001) formulated fuzzy goal programming incorporating different importance and preemtive priorities by using an additive model to maximize the sum of achievement degrees of all fuzzy goals. Lin (2004) proposed a novel weighted max–min model for fuzzy goal programming. Aköz and Petrovic (2007) proposed a new FGP method which takes into account both fuzzy goals and uncertain hierarchical levels of the fuzzy goals. Yaghoobi et al. (2008) proposed weighed additive models for fuzzy goal programming. Gupta and Bhattacharjee (2012) proposed two new methods to find the solution of fuzzy goal programming FGP problem by weighting method. Cheng (2013) presents a satisfying method which takes into account both the fuzziness and preemptive priorities of the goals for solving FGP problems with goal hierarchy. In addition to these studies, Aouni et al. (2009) and Chanas and Gupta (2002) can be referred to for the classifications in Fuzzy Goal Programming.

The goals in Fuzzy Goal Programming happen to be fuzzy goal in the three types of fuzziness (Chanas & Gupta, 2002). Coefficients in all these goals are crisp, but only the goals are fuzzy (Tiwari, Dharmar &Rao, 1986). They are:

$$fuzzy\ goal\ A: (AX)_i \lessapprox b_i \qquad (3)$$

$$fuzzy\ goal\ B: (AX)_i \gtrapprox b_i \qquad (4)$$

$$fuzzy\ goal\ C: (AX)_i \cong b_i \qquad (5)$$

where, $(AX)_i = \sum_{i=1}^{k} a_{ij}x_j$, j=1,2,…,n, "~" is a fuzzier representing the imprecise fashion in which the goals are stated. $b_i$ is aspiration level for the *i*th goal (Kim & Whang, 1998). Fuzzy goals can be defined by using different types of membership functions. The linear membership functions for the fuzzy goals given above can be seen below (Zimmermann, 1978), (Narasimhan 1980), (Hannan, 1981a).

Fig. 1 provides the membership function and the graph for the membership function for Fuzzy Goal A. There is a descending structure in Fig. 1 depending on the $\Delta_{iR}$ value.

$$\mu_i(AX) = \begin{cases} 1 & , \quad (AX)_i \le b_i \\ \frac{(b_i+\Delta_{iR})-(AX)_i}{\Delta_{iR}} & , \quad b_i \le (AX)_i \le b_i + \Delta_{iR} \\ 0 & , \quad (AX)_i \ge b_i + \Delta_{iR} \end{cases} \qquad (6)$$

Here, $\Delta_{iR}$ demonstrates the amount of acceptable deviation from $b_i$.

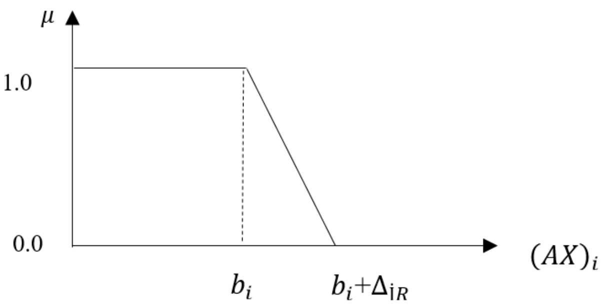


**Figure 1.** Membership function for Fuzzy Goal A.

There is an ascending membership degree in Fig. 1 depending on $\Delta_{iL}$ value. Fig. 2 shows the membership function and the graph for the membership function for Fuzzy Goal B.

$$\mu_i(AX) = \begin{cases} 1 & , \quad (AX)_i \geq b_i \\ \frac{(AX)_i-(b_i-\Delta_{iL})}{\Delta_{iL}} & , \quad b_i - \Delta_{iL} \leq (AX)_i \leq b_i \\ 0 & , \quad (AX)_i \leq b_i - \Delta_{iL} \end{cases} \tag{7}$$

Here, $\Delta_{iR}$ demonstrates the amount of acceptable deviation from $b_i$.

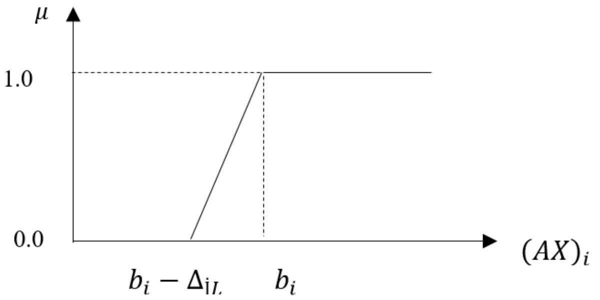


**Figure 2.** Membership function for Fuzzy B.

Fig. 3 shows the membership function and the graph for the membership function for Fuzzy Goal C. In Fig. 3, there is an ascending membership degree based on $\Delta_{iL}$ value and a descending structure based on $\Delta_{iR}$ value.

$$\mu_i(AX) = \begin{cases} 0 & , \quad (AX)_i \leq b_i - \Delta_{iL} \ or \ (AX)_i \geq b_i + \Delta_{iR} \\ \frac{(AX)_i-(b_i-\Delta_{iL})}{\Delta_{iL}} & , \quad b_i - \Delta_{iL} \leq (AX)_i \leq b_i \\ \frac{(b_i+\Delta_{iR})-(AX)_i}{\Delta_{iR}} & , \quad b_i \leq (AX)_i \leq b_i + \Delta_{iR} \end{cases} \tag{8}$$

Here, $\Delta_{iL}$ and $\Delta_{iR}$ shows the acceptable deviations from $b_i$, respectively.

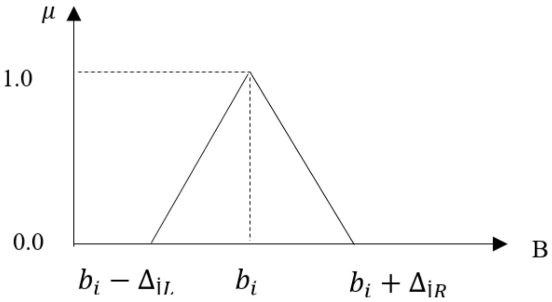


**Figure 3.** Membership function for Fuzzy Goal C

where $\Delta_{iL}$and $\Delta_{iR}$ are chosen constants of the maximum admissible violations from the aspiration level $b_i$.

### 3.1. MA approach

Also known as Chebyshev Goal Programming, Minmax Goal Programming uses $D_\infty$ metric instead of $D_1$ metric where Prioritized and Weighted Goal Programming methods are used (Romero, 1985). Minmax Goal Programming can be defined mathematically as seen below (Flavell, 1976).

$Min\ d$

Subject to; (M1.4)

$\alpha_i \frac{n_i}{k_i} + \beta_i \frac{p_i}{k_i} \leq d$

$f_i(x) + n_i - p_i = b_i$

$n_i . p_i = 0, d, n_i, p_i \geq 0 \ \ and \ i = 1, \dots, K.$

Here, $b_i$ is aspiration level for ith goal, $n_i, p_i$ are negative and positive deviations from aspiration value of ith goal, $k_i$ is the normalization constant for ith goal.

In this type of Minmax Goal Programming, maximum deviation is minimized instead of minimization of the sum of deviating variables, which is different from the weighted and prioritized structures. Goals are defined individually, and the solution is done with traditional simplex algorithm in this model. The goal function of the model is made up of the distance parameter which defines the minimization of the maximum deviation (Ignizio & Cavalier, 1994). The present study uses MA approach based on Minmax GP which was proposed by Yaghoobi and Tamiz (2007). As MA approach uses "nonsymmetric triangular membership functions", it can be regarded as an extension of the approach by Hannan (1981b), (Yaghoobi & Tamiz 2007). According to Yaghoobi andTamiz (2007), Hannan's (1981ab) approach can be considered as Minmax. MA approach is given below.

$$Max\ \lambda$$
Subject to (M1.5)
$$(AX)_i - p_i \leq b_i$$
$$\lambda + \frac{1}{\Delta_{iR}} p_i \leq 1$$
$$(AX)_i + n_i \geq b_i$$
$$\lambda + \frac{1}{\Delta_{iL}} n \leq 1$$
$$(AX)_i + n_i - p_i = b_i$$
$$\lambda + \frac{1}{\Delta_{iL}} n_i + \frac{1}{\Delta_{iR}} p_i \leq 1$$
$$\lambda, p_i, n_i \geq 0$$
$$X \in C_s$$

MA model equal weights for the fuzzy goals have been considered. Here, $n_i$ and $p_i$ are the deviated variables of ith goal, $\Delta_{iL}$ and $\Delta_{iR}$ are the acceptable left and right side deviations for ith fuzzy goal, and $b_i$ is the aspired level for ith goal.

### 3.2. De Novo Programming based on MA approach

Positive and negative ideal solution values of each goal function are considered as aspiration levels in this study in order to provide solutions based on MA approach. For the goal functions to turn into goals, the following method is applied. A maximization directed goal function must be at least $Z_k^*$, aminimization directed goal function must be $W_s^*$ at most (in another sense, "=" can be used for each goal). Following organizations are done for goal function types. In addition, the deviation values are taken as $(Z_k^* - Z_k^-)$ for maximization directed goals and as $(W_s^- - W_s^*)$ for minimization directed goals considering the fact that goal function may have different units. Maximization directed goal function can be written by using "≥ " in the goal type below.

$$Z_k(x) + n_i \geq Z_k^* \quad (9)$$

$$\lambda + \frac{1}{(Z_k^* - Z_k^-)} n_i \leq 1 \quad (10)$$

Here, $(Z_k^* - Z_k^-) = \Delta_{ZL}$.

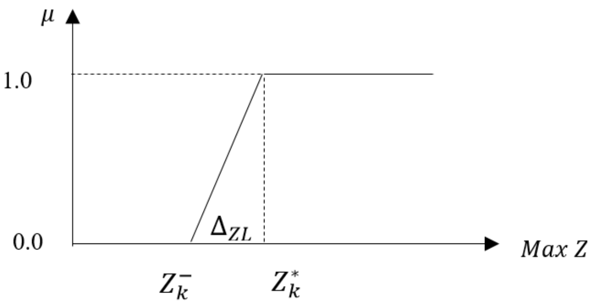


**Figure 4.** Membership function for Max. goal

The goal transformed into "$\leq$ " type for minimization directed goal function;

$$W_s(x) - p_i \leq W_s^* \tag{11}$$

$$\lambda + \frac{1}{(W_s^- - W_s^*)} p_i \leq 1 \tag{12}$$

Here $(W_s^- - W_s^*) = \Delta_{WR}$.

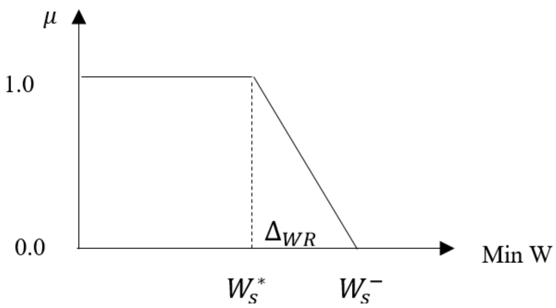


**Figure 5.** Membership function for Min. goal

For the goal transformed into "=" type

In case when the targets are transformed into "=" goal type for maximization and minimization directed goal function, positive ideal solutions cannot be used. Therefore, the decision maker has to define triangular membership function.

$$Z_k(x) + n_i - p_i = b_k \tag{13}$$

$$\lambda + \frac{1}{\Delta_{iL}} n + \frac{1}{\Delta_{iR}} p_i \leq 1 \tag{14}$$

and

$$W_s(x) + n_i - p_i \leq b_s \tag{15}$$

$$\lambda + \frac{1}{\Delta_{iL}} n + \frac{1}{\Delta_{iR}} p_i \leq 1 \tag{16}$$

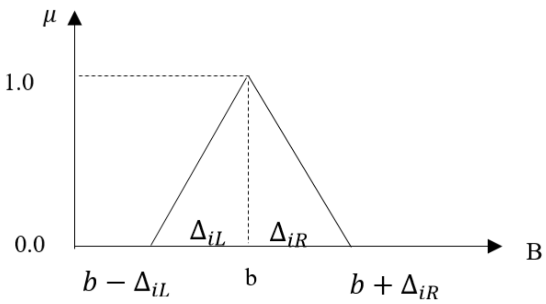


**Figure 6.** Membership function for Fuzzy Budget

The proposed solution includes the budget constrain in the De novo hypothesis as a target. In this regard, the budget target can be "≤ " or "=" type based on the decision maker's demands. If the budget target is defined at ≤ " type, the membership function must be defined based on the minimization goal function. If the budget target is "=", it must be defined as triangular membership function. In this study, the fuzzy budget target is accepted as "=" type and organized as seen below.

$$B(x) + n_i - p_i = B \quad (17)$$

$$\lambda + \frac{1}{\Delta_{BL}} n_i + \frac{1}{\Delta_{BR}} p_i \leq 1 \quad (18)$$

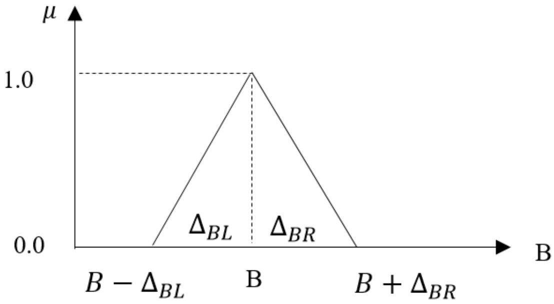


**Figure 7.** Membership function for Fuzzy budget

According to the organizations above, the solution of a De Novo Programming problem based on MA approach is given below.

$Max\ \lambda$

Constrains; (M1.6)

$$Z_k(x) + n_i \geq Z_k^*$$
$$\lambda + \frac{1}{(Z_k^* - Z_k^-)} n_i \leq 1$$
$$W_s(x) - p_i \leq W_s^*$$
$$\lambda + \frac{1}{(W_s^- - W_s^*)} p_i \leq 1$$
$$B(x) + n_i - p_i = B$$
$$\lambda + \frac{1}{\Delta_{BL}} n_i + \frac{1}{\Delta_{BR}} p_i \leq 1$$
$$\lambda, p_i, n_i \geq 0$$
$$X \in C_s$$

Here,

$(Z_k^* - Z_k^-) = \Delta_{ZL},\ (W_s^- - W_s^*) = \Delta_{WR}$.

$Z_k^*$:Positive ideal solution value for max. $Z_k(x)$

$Z_k^-$:Negative ideal solution value for max. $Z_k(x)$,

$W_s^*$: Positive ideal solution value for min. $W_s(x)$,

$W_s^-$:Negative ideal solution value for min. $W_s(x)$,

$B(x)$: Budget function,

$B$:Total budget,

$\Delta_{BL}$: Acceptable amount of subtraction from the budget,

$\Delta_{BR}$: Acceptable amount of subtraction from the budget.

## 4. Application.

Table 1 displays the resource utility amounts and resourse unit prices for the three different wooden handcraft products manufactured by the business.

| | Wood Table | Wood Coffee Table | Wood Crate | Resource Unit Price ($) |
|---|---|---|---|---|
| **Leather ($m^2$)** | 4 | 5 | 3 | 3 |
| **Wood ($m^3$)** | 7 | 6 | 8 | 3.5 |
| **Labor (h)** | 5 | 8 | 7 | 4.2 |
| **Machine Use Period (h)** | 3 | 4 | 2 | 5 |

**Table 1.** Resource Use Amounts and Resource Unit Prices

The maximum manufacturing capacity of the business is 106 units a week.Based on the previous information, the first and second products are demanded weekly as 35 and 48 at most, respectively, and the third product is demanded at least 21 units. The business has a budget of $7410 for the resources needed for production, and it is able to make a decrease on $400 or an increase of $450. The business would like to determine the amount of each resource needed for the maximum capacity. The cost of each product is $100, $92, and $85, respectively, and the profit per unit is $68, $38, and $47, respectively. Multiobjective Linear Programming problem is organized as follows based on the information.

$$
\begin{array}{l}
Max\ Z = 68x_1 + 38x_2 + 47x_3 \\
Min\ W = 100x_1 + 92x_2 + 85x_3 \\
\text{Subject to} \\
4x_1 + 5x_2 + 3x_3 = b_1 \\
7x_1 + 6x_2 + 8x_3 = b_2 \\
5x_1 + 8x_2 + 8x_3 = b_3 \\
3x_1 + 4x_2 + 2x_3 = b_4 \\
x_1 + x_2 + x_3 \leq 106 \\
x_1 \leq 35 \\
x_2 \leq 48 \\
x_3 \geq 21 \\
x_1, x_2, x_3 \geq 0\ and\ integer.
\end{array}
\tag{P1}
$$

If (P1) is organized according to De Novo Programming problem;

$$
\begin{array}{l}
Max\ Z = 68x_1 + 38x_2 + 47x_3 \\
Min\ W = 100x_1 + 92x_2 + 85x_3 \\
\text{Subject to} \\
72.5x_1 + 89.6 + 76.4x_3 \cong 7410 \\
x_1 + x_2 + x_3 \leq 106 \\
x_1 \leq 35 \\
x_2 \leq 48 \\
x_3 \geq 21 \\
x_1, x_2, x_3 \geq 0\ and\ integer.
\end{array}
\tag{P2}
$$

acquired. In order to determine the positive and negative ideal solutions of the objective functions in (P2), we use the exact sum of the budget value as $7410. When (P2) is solved in this regard, $I^* = \{4974; 8138\}\ ve\ I^- = \{4086; 8678\}$ is acquired. The objective functions and the fuzzy goals of the budget based on the aforementioned information can be seen below.

$$Max\ Z(x): 68x_1 + 38x_2 + 47x_3 \gtrapprox 4974 \tag{19}$$

$$Min\ W(x) = 100x_1 + 92x_2 + 85x_3 \lessapprox 8138 \tag{20}$$

$$B(x) = 72.5x_1 + 89.6 + 76.4x_3 \cong 7410 \tag{21}$$

Figure 8 provides the linear membership function for (19). Here, $\Delta_{ZL}= 888$ .

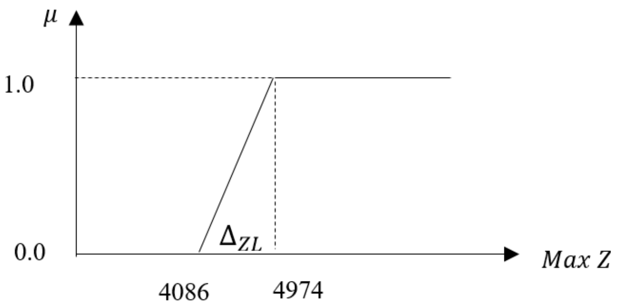


**Figure 8.** Linear membership function for Max Z.

$$\mu_Z(x) = \begin{cases} 1 & , \quad Z(x) \geq 4974 \\ \frac{Z(x)-4086}{888} & , \quad 4086 \leq Z(x) \leq 4974 \\ 0 & , \quad Z(x) \leq 4086 \end{cases} \tag{22}$$

The linear membership function for (20) is displayed in Figure 9. Here, $\Delta_{WR}= 540$ .

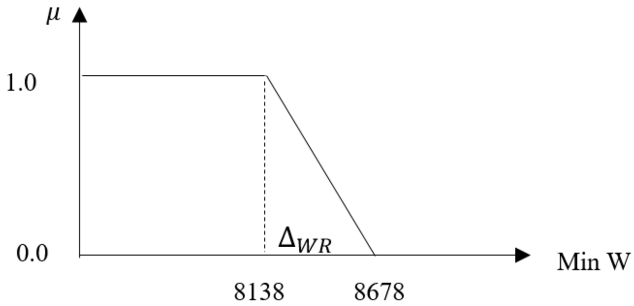


**Figure 9.** Linear membership function for Max W.

$$\mu_W(x) = \begin{cases} 1 & , \quad W(x) \leq 8138 \\ \frac{8678-W(x)}{540} & , \quad 8138 \leq W(x) \leq 8678 \\ 0 & , \quad W(x) \geq 8678 \end{cases} \tag{23}$$

The triangular membership function for (21) is displayed in Figure 10. Here $\Delta_{BL}= 400$ and $\Delta_{BR}= 450$ .

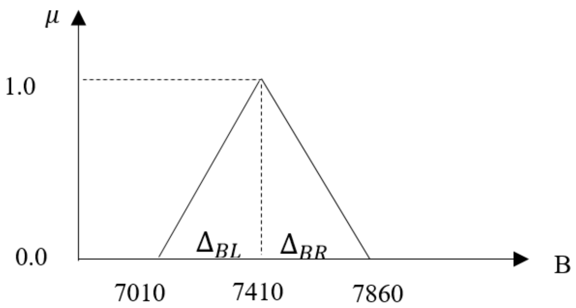


**Figure 10.** The triangular membership function for the budget.

$$\mu_B(x) = \begin{cases} 0 & , \quad B(x) \le 7010 \ or \ B(x) \ge 7860 \\ \frac{B(x)-7010}{400} & , \quad 7010 \le B(x) \le 7860 \\ \frac{7860-B(x)}{450} & , \quad 7410 \le B(x) \le 7860 \end{cases} \tag{24}$$

(P2) is organized as seen below according to (M1.6) as proposed considering the equations (22), (23), and (24).

$Max\ \lambda$
Subject to (P3)

$$68x_1 + 38x_2 + 47x_3 + n_1 \ge 4974$$
$$\lambda + \frac{1}{\Delta_{ZL}} n_1 \le 1$$
$$100x_1 + 92x_2 + 85x_3 - p_2 \le 8138$$
$$\lambda + \frac{1}{\Delta_{WR}} p_2 \le 1$$
$$72.5x_1 + 89.6 + 76.4x_3 + n_3 - p_3 = 7410$$
$$\lambda + \frac{1}{\Delta_{BL}} n_3 + \frac{1}{\Delta_{BR}} p_3 \le 1$$
$$x_1 + x_2 + x_3 \le 106$$
$$x_1 \le 35$$
$$x_2 \le 48$$
$$x_3 \ge 21$$
$$n_1, p_{2,} n_3, p_3 \ge 0, x_1, x_2, x_3 \ge 0 \ and \ integer.$$

The information acquired from the solution of (P3) is given in Table 2. Table 2 displays decision variables, the values of each goal function, membership degrees of each goal function, and the membership degree of the whole model.

| **Decision Variables** | $Z(x)$ | $W(x)$ | $B(x)$ |
|---|---|---|---|
| $x_1$ | 9 | 9 | 9 |
| $x_2$ | 0 | 0 | 0 |
| $x_3$ | 87 | 87 | 87 |
| | | | |
| **Target Function Value** | 4701 | 8295 | 7299.3 |
| **Membership Function Value** | 0.692568 | 0.709259 | 0.72325 |

**Table 2.** The Results of the Proposed Model

The solution of (P3) gives $\lambda = 0.692568$ as the result. It is the membership degree of the whole model, which means all the goals are met at the membership degree. Additionally, each goal function and budget goal were realized at the same decision variable value. According to the De novo solution proposal based on MA approach, the profit goal (target) is realized as \$4701, and the cost goal (target) is realized as \$8295. On the other hand, a total of \$7299.3 from the budget is spent for the resources to reach these production values. Table 3 presents the needed amount of resources based on the information above.

| **Resources** | **Proposed Resource Amount** $(b_i)$ |
|---|---|
| $b_1$: Leather ($m^2$) | 297 |
| $b_2$: Wood ($m^3$) | 759 |
| $b_3$: Labor (h) | 741 |
| $b_4$: Machine Use Period (h) | 201 |
| Budget | 7664.7 |

**Table 3.** Resource Amounts based on the proposed solution.

The total budget is defined as \$7664.7 in terms of the information in Table 3. A surplus is defined for the resource 3 due to the total production capacity and demand for the products. It makes for \$364.4. The total budget, therefore, is \$7299.3+\$365.4=\$7664.7. When the given (P2) is organized based on the classical minmax goal programming (M1.4);

$$
\begin{aligned}
& Min\ d \\
& \text{Subject to} \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{(P4)} \\
& Max\ Z(x): 68x_1 + 38x_2 + 47x_3 + n_1 = 4974 \\
& \frac{n_1}{888} \leq d \\
& Min\ W(x) = 100x_1 + 92x_2 + 85x_3 - p_2 = 8138 \\
& \frac{p_2}{450} \leq d \\
& B(x) = 72.5x_1 + 89.6 + 76.4x_3 + n_3 - p_3 = 7410 \\
& \frac{n_3}{400} + \frac{p_3}{450} \leq d \\
& x_1 + x_2 + x_3 \leq 106 \\
& x_1 \leq 35 \\
& x_2 \leq 48 \\
& x_3 \geq 21 \\
& x_1, x_2, x_3 \geq 0\ and\ integer.
\end{aligned}
$$

when the goal function of (P2) is organized. The profit target is $p_1 = 0$ for (P4) because ($Z(x)$) cannot exceed its own ideal solution value. The cost target is $n_2 = 0$ because ($W(x)$) would not go below its own ideal solution. The normalization constants for both targets are 888 and 450, respectively. In addition, the budget target is determined as 7410, and tolerance values which were \$400 below or \$450 above this target are used as normalization constant. The results acquired from the solution of (P4) are given in Table 4.

| **Decision Variables** | $Z(x)$ | $W(x)$ | $B(x)$ |
|---|---|---|---|
| $x_1$ | 9 | 9 | 9 |
| $x_2$ | 0 | 0 | 0 |
| $x_3$ | 87 | 87 | 87 |
| | | | |
| Target Function Value | 4701 | 8295 | 7299.3 |
| Distance Degree to the Positive Ideal Solution | *0.307398* | 0.290764 | 0.27675 |

**Table 4.** Minmax Goal Programming solutions

It is defined as $d = 0.307398$ with the solution of (P4). This value is the same as the degree of the distance to the positive ideal solution of $Z(x)$. Additionally, all the goals are realized based on the same decision variables.

## 5. Conclusion

In this study, the positive ideal solution values of profit and cost goals are transformed into targets by being assigned as aspiration levels for goals. In terms of MA approach, the difference between the positive and negative ideal solutions of the goals are defined as tolerance values of acceptability. The budget constrain in (P2) is transformed into a goal whose tolerance values were set by the decision maker. In terms of Minmax, positive and negative ideal solution differences are assigned as normalization constants to profit and cost targets. The tolerance values of the budget are used as normalization constant for deviations for the budget target.

According to the aforementioned information, it is seen that decision variable values and goal (target) function values are the same as seen in Table 2 and 4. Therefore, the variables which are acquired from minmax Goal Programming are the same as the resource amounts provided by the proposed approach as seen in Table 3. On the other hand, Zeleny (1982) and Lee & Li (1997) provided detailed explanations on "$\lambda = 1 - d$" although membership degree and degree of distance to ideal are perceived to be different. While Minmax Goal Programming aims to minimize the distance to positive ideal solutions, the fuzzy solution aims to maximize the proximity to the positive ideal solutions. In this regard, MA approach and Minmax Goal Programming provide the same result. Future studies may provide investigations from different viewpoints by also using Hannan (1981ab) approach, which is the origin on MA approach, in addition to the minmax based models for optimal system design.

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