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Rank reversal on entropy-based weighting methods

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Abstract

Entropy is an important criteria weighting measure used in decision making. There are different forms of entropy that are used to measure the inter criterion contrast intensity. In this study, we defined various entropy and diversity measures as criteria weighting approach in MCDM. We compared the approaches in terms of the rank reversal phenomenon by conducting a simulation study according to the framework we established. In addition, we compared these weighting approaches in terms of their dispersion characterization in an illustrative case. The Gini-Simpson index is the foremost index among these approaches, which is more persistent to rank reversal, less sensitive to distribution of domain and outputs more acceptable weightings.

Keywords Multiple criteria analysis, Criteria weighting, Shannon entropy, Gini-Simpson diversity, Yager entropy, Rao entropy, Rank reversal

Jel Codes C15, C44, C61

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1. Introduction

The problem involving the ranking of a certain number of alternatives by evaluating them over a specific set of criteria is called Multi-Criteria Decision Making (MCDM) problem. To handle MCDM problems and assist the decision-maker, there are numerous methods that have been developed and applied various areas. The state of art methods can be tracked down in assorted fields such as; renewable energy (Lak Kamari et al., 2020), business analytics (Yalcin et al., 2022), heritage buildings (Nadkarni & Puthuvayi, 2020), sustainability engineering (Stojčić et al., 2019), corporate sustainability (Chowdhury & Paul, 2020), failure mode and effect analysis (Liu et al., 2019), sustainable supply chain management (Paul et al., 2021), sustainable manufacturing (Jamwal et al., 2020; Pelissari et al., 2021), construction (Zhu et al., 2021), transportation (Yannis et al., 2020), financial modelling (Almeida-Filho et al., 2020), intrusion detection systems (Alamleh et al., 2022), supply chain in the industry 4.0 (Unhelkar et al., 2022). Bibliometric studies are also carried out regarding the usage areas of MCDM (Bragge et al., 2012).

MCDM approaches support decision makers by taking an analytical role in the decision-making phase when there are multiple criteria and alternatives in a decision problem, in the light of these criteria. In this case, the most basic structure affecting the overall ranking is the criteria weight vector (Zardari et al., 2015).

Criteria weights can be determined subjectively by experts in the field, or they can be obtained objectively based on the decision matrix alone. The second approach is often preferred due to expert's possible bias or inaccuracy in decision-making regarding the importance of the criteria, as it can eliminate the both of these flaws. Odu (2019) also stated that decision-makers usually struggle to assign criteria weights with pinpoint accuracy.

For these previously mentioned reasons, the use of objective criteria weighting approaches in MCDM has become widespread. The Entropy method, which is based on information theory has been one of the first addresses referenced at this point (Sidhu et al., 2022). The Entropy method which basically uses Shannon (1948)'s Entropy, measures the degree to which criteria distinguish alternatives most.

The advantages of the Entropy method are that it provides a simple, uncomplicated, and unbiased approach to calculating the weights of the criteria by taking into account the contrast intensity (Kumar et al., 2021), while the shortcomings of the approach are as follows; the Entropy method has been criticized for producing distorted results when there is a zero value in the decision matrix (Zhu et al., 2020), it can assign weights to criteria with large differences between the largest and smallest importance levels (e.i. hypersensitivity), and even in some extreme cases, a criterion with a zero importance value can be found, which is not acceptable in the context of MCDM theory (Wang & Luo, 2010).

In MCDM theory, there is another prominent debate is concerning practitioners about the independence of the alternatives ranking's from adding or removing of an alternative which is namely Rank Reversal (RR) phenomenon (Bączkiewicz et al., 2021). Decision makers expect criteria weighting methods to not cause this problem or at least to limit it.

The RR problem was introduced in MCDM discussing through the Analytical Hierarchy Process (AHP) and an approach was proposed to overcome this problem (Belton & Gear, 1983). However, it was later stated that this approach could not prevent RR either (Saaty & Vargas, 1984). Since then, the RR problem has been frequently studied, both with related to AHP (Eskandari & Rabelo, 2007; Maleki & Zahir, 2012) and generally in decision making (Ayrim et al., 2018; Ceballos et al., 2018; Farias Aires & Ferreira, 2018; Nazari-Shirkouhi et al., 2011; Saaty & Ergu, 2015; Wang & Luo, 2009).

While some authors proposed new MCDM techniques to prevent RR (Mousavi-Nasab & Sotoudeh-Anvari, 2018; Mufazzal & Muzakkir, 2018; Munier, 2016) and some others compared existing methods in terms of RR tendency (Hinduja & Pandey, 2021; Keshavarz-Ghorabaee et al., 2018). Additionally there were efforts to explore the RR resistantancy of normalization methods (Aytekin, 2021) and to determine local criteria weights using MCDM approach resistant to RR (Bączkiewicz et al., 2021)

It is acknowledged that one of the reasons behind the RR is the criteria weights (Farias Aires & Ferreira, 2018). The obtained weight vector by the Entropy method, which is based on the dispersion of performance scores of alternatives on each criterion, is inevitably affected by the inclusion or exclusion of an alternative.

RR occurring rates are taken into account as an important measure in comparing the effectiveness of MCDM techniques (Keshavarz-Ghorabaee et al., 2018). However, RR comparative analyzes are mostly performed between ranking MCDM methods (Farias Aires & Ferreira, 2019). In the literature, there is a great lack of comparing the effectiveness of objective criteria weighting methods on the basis of RR.

The aim of the study is to define the use of different types of entropy and diversity indices in criteria weighting and to perform a simulation-based comparative efficiency analysis on RR. For this, we paired them with two basic ranking approaches, the Weighted Sum Method (WSM) and the Weighted Product Method (WPM). Next, we investigated how much RR occurrences ratios the criteria weights cause in the ranking methods. Additionally the dispersion characterizations of each approach were explored to identify their responses in different cases.

Our main motivation and research problem is to seek answers to the question of which entropy functions are more resistant to RR and can get more acceptable results in criteria weighting, which are the main problems in MCDM. So in this study we have research questions as "How do different entropy functions handle rank reversal, and which are more robust against it?" and "Which method entropy function provides sufficient contrast intensity while also yielding less hypersensitivity?"

The rest of the paper is organized as follows. In Section 2 we introduced various entropy-based approaches in criteria weighting. The simulation-based comparative analysis was conducted to acquire the efficiency of each entropy-based weighting method in terms of RR in section 3. Next section is organized to explore the dispersion characterization of each entropy-based weighting method. The conclusion which includes the overview of entropy-based weighting methods and criteria weighting related RR is the last part of the study.

2. Entropy-based Methods

Shannon entropy is known as the entropy type used in the Entropy method in the MCDM literature. Apart from the classical Entropy method, entropy is generally used in MCDM with the fuzzy-type of Shannon entropy (Liu et al., 2020; Lotfi & Fallahnejad, 2010) or fuzzy and similar derivatives of cross-entropy (Khalaj et al., 2019; Li et al., 2020; Wei, 2016).

We, on the other hand, made comparisons between methods by adapting different types of entropies to MCDM as an alternative to Shannon entropy, as follows.

2.1. Shannon Entropy Method

One of the most popular objective criteria weighting methods was defined by Hwang & Yoon (1981) based on the Shannon (1948) Entropy (SE) in Eq. 1

$$H = -\sum_{i=1}^{n} p_i \ln p_i \tag{1}$$

The entropy method by Hwang & Yoon (1981) is described as follows;

Suppose the decision matrix $G = \|g_{ij}\|_{n*m}$ includes n number of alternatives with m number of criteria set where g_{ij} denotes the score of the alternative i^{th} based on criterion j^{th}. The entropy method normalizes the matrix via Eq. 2

$$z_{ij} = \frac{g_{ij}}{\sum_{i=1}^{n} g_{ij}} \tag{2}$$

The (Shannon) entropy of criterion j^{th} is defined in Eq. 3;

$$e_j = \frac{-1}{\ln(n)} \sum_{i=1}^{n} d_{ij} \ln d_{ij} \tag{3}$$

The importance of criterion jth can be obtained by employing Eq. 4 as follows;

$$we_j = \frac{1 - e_j}{m - \sum_{i=1}^{m} e_j} \tag{4}$$

2.2. Gini-Simpson diversity Method

The development of Gini-Simpson diversity (GSD) index goes back to Gini (1912) (see Eq. 5).

$$D = 1 - \sum_{i=1}^{n} p_i^2 \tag{5}$$

The GSD is quite popular in various field such as portfolio optimization (Aksaraylı & Pala, 2018), urban water management (Gonzales & Ajami, 2017), and health (Rai & Kim, 2020). GSD had some implementations in MCDM concept within fuzzy framework (Kumar & Kumar, 2021; 2022; Ma, 2019). However, a method definition is required for the GSD in criteria weighting to be suitable for comparison. We described the criteria weighting via GSD as follows;

The normalization process of the decision matrix $G = \|g_{ij}\|_{n*m}$ has to be same with SE as in Eq. 2. Then the GSD of criterion j^{th} can be acquired by Eq. 6

$$gs_j = \sum_{i=1}^{n} z_{ij}^2 \tag{6}$$

The criteria weights according to GSD can be evaluated by Eq. 7 as follows;

$$wgs_j = \frac{gs_j}{\sum_{j=1}^{m} gs_j} \tag{7}$$

2.3. Yager Entropy Method

Yager (1988) introduced Ordering Weighted Averaging (OWA) operators as an aggregation method for information processing in MCDM, and then Yager (1995) defined Yager entropy (YE) to characterize the dispersion property of OWA operators as follows;

$$Y = -\sum_{i=1}^{n} \left| p_i - \frac{1}{n} \right| \tag{8}$$

Other than OWA related works (see Emrouznejad & Marra (2014), Yager entropy has become quite popular in various fields such as portfolio optimization (Yue & Wang, 2017), image segmentation (Ghosh et al., 2010), and agriculture (Adeel et al., 2020).

To compare Yager entropy in criteria weighting with SE and GSD adequately, we also employed Eq. 2 in normalizing decision matrix. Then, the relative importance of criterion jth and its overall importance are evaluated via Eq. 9 and Eq. 10, respectively.

$$y_j = -\sum_{i=1}^{n} \left| z_{ij} - \frac{1}{n} \right| \tag{9}$$

$$wy_j = \frac{y_j}{\sum_{j=1}^{m} y_j} \tag{10}$$

2.4. Rao Entropy Method

Rao (1982) introduced quadratic entropy which is known as Rao Entropy (RE) as,

$$QE = \sum_{i=1}^{S-1} \sum_{k=i+1}^{S} d_{ik} p_i p_k \tag{11}$$

where d_{ik} can be acquired by Eq. 12

$$d_{ik} = \frac{1}{m} \sum_{j=1}^{m} \left(X_{ij} - X_{kj} \right)^2 \tag{12}$$

Rao's Entropy became a useful tool to assess partitioning and dispersion in various field such as portfolio optimization (Carmichael et al., 2017), patent document analysis (Khachatryan & Muehlmann, 2019), and environmental heterogeneity research (Doxa & Prastacos, 2020).

We defined assessing criteria weight via Rao Entropy in Eq. 13 and Eq. 14 as follows;

$$r_j = \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} d_{ik} z_{ij} z_{kj} \tag{13}$$

$$wr_j = \frac{r_j}{\sum_{j=1}^{m} r_j} \tag{14}$$

where d_{ik} and z_{ij} can be obtained by Eq. 15 and Eq. 2, respectively.

$$d_{ik} = \frac{1}{m}\sum_{j=1}^{m}\left(z_{ij} - z_{kj}\right)^2 \tag{15}$$

3. Comparative Analysis

In this section, we performed comparative analysis between four types of entropy method in terms of various RR cases in ranking with WSM and WPM. Additionally, we analyzed the alteration of ranking order in criteria weights in RR instances. To examine in detail, we generated random problem examples using same outlines in parallel with Farias Aires & Ferreira (2019) and Baykasoğlu & Ercan (2021), but by making some changes like increasing number of alternatives and criteria classes from 4 to 5 (calling them as low, lower-middle, middle, upper-middle, high), expanding interval range from [0-200] to [0-1000] and augmenting replications number from 100 to 1000 in the input parameters as follows;

- Number of Alternatives (n): 4 (low), 8 (lower-middle), 12 (middle), 16 (upper-middle), 20 (high).
- Number of Criteria (m): 4 (low), 8 (lower-middle), 12 (middle), 16 (upper-middle), 20 (high).
- Scores of the alternatives for each criterion: randomly generated by employing a uniform distribution in the interval [0–1000];
- Number of replications; 1000 instances for each combination, thence 25000 different decision problems are generated.

Within this concept, the entropy based weighting methods (SE, GSD, YE, RE) compared in WSM and WPM through the five different types of RR as follows;

Type 1: RR occurs if the alternative rank order is altered with including an irrelevant alternative to the problem. See, Verly & Smet (2013) and Cinelli et al. (2014). In this case we added an alternative which might yields changing in criteria weights to the problem and check out if the alternative rank order is also changed.

Type 2. RR occurs if the best alternative is changed with including an irrelevant alternative to the problem. See, Farias Aires & Ferreira (2019) and Baykasoğlu & Ercan (2021). In this case we added an alternative which might yields changing in criteria weights to the problem and check out if the best alternative is also changed.

Type 3. RR occurs if the alternative rank order is altered with incorporating an identical copy of an alternative to the problem. See, Baykasoğlu & Ercan (2021). In this case we added an alternative which might yields changing in criteria weights to the problem and check out if the alternative rank order is also changed.

Type 4: RR occurs if the alternative rank order is altered with excluding an irrelevant alternative from the problem. See, Verly & Smet (2013) and Cinelli et al. (2014). In this case we removed an alternative which might yields changing in criteria weights from the problem and check out if the alternative rank order is also changed.

Type 5. RR occurs if the best alternative is changed with discarding an irrelevant alternative from the problem. See, Farias Aires & Ferreira (2019) and Baykasoğlu & Ercan (2021). In this case we dropped an alternative which might yields changing in criteria weights to the problem and check out if the best alternative is also changed.

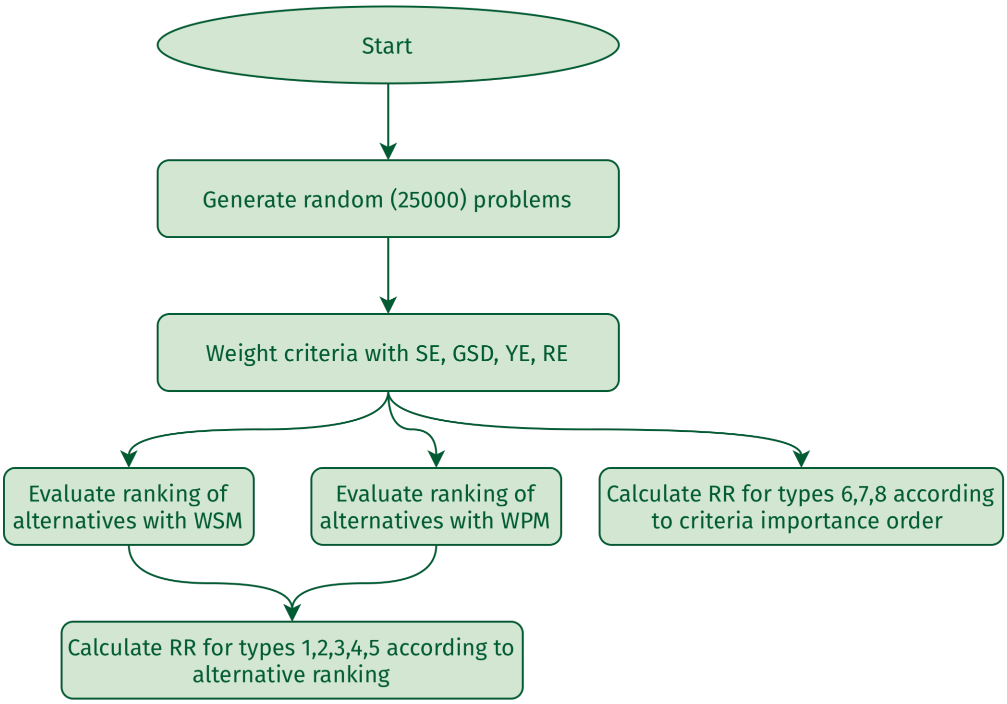
Additionally we explored RR phenomenon in criteria ranks with defining 3 extra types of RR as follows;

Type 6. RR occurs when the addition of an irrelevant alternative causes any ranking change in criteria importance order.

Type 7. RR occurs if the criteria rank order is altered with incorporating an identical copy of an alternative to the problem.

Type 8. RR occurs when the extraction of an irrelevant alternative causes any ranking change in criteria importance order.

In this study, we solved 25000 decision problems and the RR conditions in WSM and WPM and criteria were evaluated simultaneously. The flowchart of the study represented as follows.



3.1. Comparing entropy based methods for rank reversal problem in WSM

The WSM method is a pioneer in decision-making and one of the mostly ever used approaches in MCDM problems. The total score of an alternative S_i can be obtained by WSM (Fishburn, 1967) as follows,

$$S_i = \sum_{j=1}^{m} w_j z_{ij}, \quad i = 1, ..., n \tag{16}$$

where z_{ij} are the normalized matrix elements and can be acquired by Eq. 2.

In Table 1, we presented RR average ratios (Type 1-5 used as T1-T5) according to each weighting methods in WSM The comma-separated data in the table are ordered from the lowest to the highest number of alternatives. For example, the 1st and 5th data points are the RR percentages obtained with the low (n=4) and high (n=20) number of alternatives, respectively.

Accordingly first data points present the RR results for the decision problems which the numbers of alternatives are considerably low. When the number of alternatives is low, as the number of criteria increases, the RR type that increases the most is Type 3, while Type 2 is the least observed one in general. As an exception to least observed type of RR, the results obtained with SE can be seen (see Type 5) when the number of criteria is low, but as the number of criteria increases, a similar situation prevails for SE (see Type 2) as with the others.

In second data points, the RR results for the decision problems which the numbers of alternatives are considerable in the lower-middle category are provided. When the number of alternatives is lower-middle and the number of criteria is low the highest RR occurrences are observed in Type 1, while with the increase in the number of criteria the Type 3 has become be observed more. The least observed type of RR is actualized as Type 2.

Third data points present results for the decision problems with number of alternatives are considerable in the middle category. While Type 1, 3, and 4 have high occurrences ratios even with low number of criteria, they all goes up to higher levels with the increase in number of criteria. Contrastingly, the Type 2 and 5 which both can be observed similar each other and lesser than others, are not ascended rapidly with the increase in the number of criteria.

In forth data points, we present the results with number of alternatives are considerable in the upper-middle category. While the rate of observation of Types 1, 3 and 4 was at least 2 out of 3 even for a low number of criteria, this rate became around 90% when the number of criteria increased. The rate of increase in the incidence of Types 2 and 5, when the number of criteria increased, decreased considerably. Fifth data points present results for the decision problems with the maximum number of alternatives. While the rate of observation of Types 1, 3 and 4 was at least 3 out of 4 even for a low number of criteria, this rate became around 95% when the number of criteria increased. Contrastingly, Types 2 and 5 are not observed more than 7% even when the number of criteria is at maximum.

In general, The GSD yielded better results than any of the other methods in every of RR types with the exception of Type 3 in which RE outperformed all. This is due to RE method consists of distances between each alternative as a multiplication element and in Type 3 where a copy of an alternative would have zero distance with the original one. Therefore, both of these criteria weights may have lower r_j values and their share in weighting would be diminished. Accordingly adding a copied alternative (See Type 3) would be a lesser problem for RE. Another general inference is that as the number of alternatives increases, the occurrences of Type 1, 3 and 4 increase, while Type 2 and 5 decrease.

Table 1. RR ratios of each entropy based methods in WSM

		SE	GSD	YE	RE
m=4	T1	31,59,76,81,85	25,50,65,78,81	31,56,73,83,84	33,63,78,86,92
	T2	12,9,7,6,5	9,7,5,4,3	10,9,7,4,3	12,10,8,5,4
	T3	31,59,74,83,86	30,54,69,74,83	33,59,73,82,84	30,43,55,67,76
	T4	23,56,74,82,86	16,46,61,72,80	20,52,69,78,85	23,58,75,84,89
	T5	10,12,8,7,5	9,7,4,3,2	10,9,6,4,4	12,12,7,5,4
m=8	T1	41,74,86,91,94	28,60,76,83,89	36,66,83,89,94	39,68,83,91,95
	T2	16,11,10,7,7	11,7,6,3,3	12,9,7,6,5	15,10,8,6,5
	T3	49,75,89,94,97	40,66,77,88,92	45,72,86,92,96	34,52,67,77,81
	T4	28,66,86,91,95	22,53,72,79,88	27,62,79,86,92	27,63,80,90,95
	T5	14,13,9,10,7	12,8,5,4,3	14,10,7,6,5	15,13,8,7,6
m=12	T1	45,78,90,95,97	34,61,77,87,93	38,69,86,92,95	40,72,87,93,97
	T2	15,14,11,9,7	13,9,7,5,4	14,11,8,6,5	16,12,8,8,6
	T3	56,84,91,96,97	48,70,82,91,93	55,78,89,95,96	41,57,70,81,84
	T4	28,67,86,95,96	23,54,72,84,90	26,61,81,91,94	29,70,85,92,96
	T5	16,13,11,9,7	14,10,7,4,4	15,11,9,8,8	17,14,10,8,6
m=16	T1	47,78,91,97,98	36,61,78,86,92	42,70,86,93,96	45,80,87,95,96
	T2	16,12,10,9,7	12,9,6,4,4	14,12,6,7,6	15,13,9,8,6
	T3	65,86,94,97,99	55,74,85,91,95	59,83,92,94,98	45,57,70,81,87
	T4	33,70,87,94,97	25,57,75,85,91	29,64,82,92,95	33,70,89,93,96
	T5	17,14,10,8,6	14,10,7,4,4	16,12,9,7,5	20,15,10,7,7
m=20	T1	47,78,91,96,98	35,64,77,88,90	42,71,85,93,96	45,80,90,95,97
	T2	17,12,11,8,6	13,10,7,6,4	15,10,8,7,6	15,15,11,8,6
	T3	68,90,96,98,98	60,80,87,92,95	65,86,92,96,98	47,62,76,83,87
	T4	34,74,88,96,98	26,57,74,84,90	29,66,81,93,96	34,74,89,93,97
	T5	21,17,11,8,7	15,10,7,7,3	17,11,8,7,6	18,15,10,10,6

We also evaluated the averages for each type of RR and overall average RR according to weighting methods. The results are provided in Figure 1. In WSM, while the observation rate of Type 2 and 5 are around 10%, the occurrences of Type 1, 3 and 4 are very high as about 80%. According to results provided Table 1 and also overall averages, the GSD is the most robust entropy based weighting scheme for WSM, while SE is the least. Figure 2 ranks the entropy functions in terms of resistance to RR in WSM. In these rankings that are made for different types of RRs with alternatives and criteria of different sizes, for example A4T1K8 indicates that the ranking is made for type 1 RR with 4 alternatives and 8 criteria.

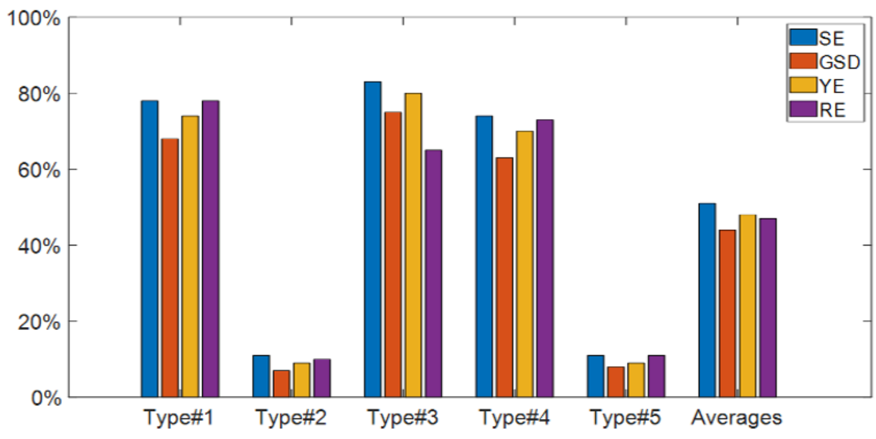


Figure 1. RR averages according to each weighting method in WSM

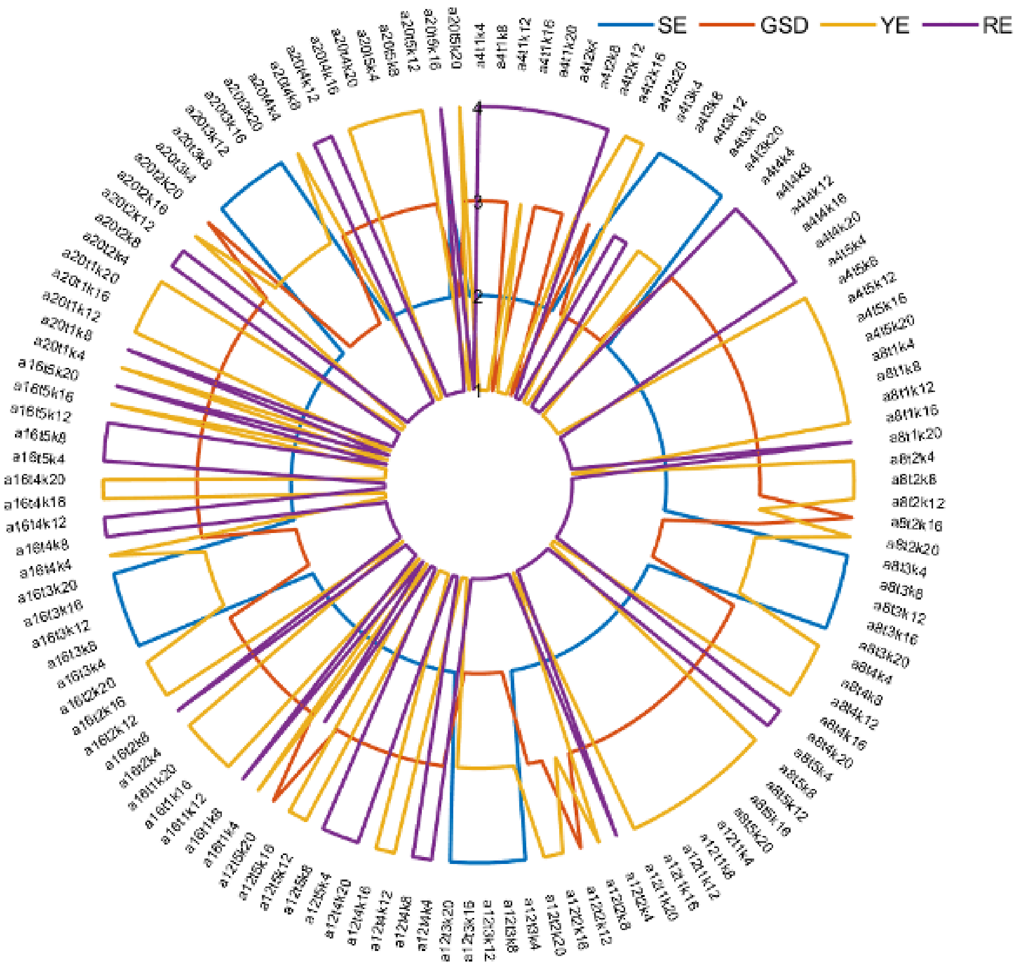


Figure 2. RR resistance ranks of weighting methods in WSM

3.2. Comparing entropy based methods for rank reversal problem in WPM

The WPM is a foremost ranking method in MCDM and the overall utility score of alternatives can be acquired by Eq. 17 as follows (Triantaphyllou & Mann, 1989; Zavadskas et al., 2012).

$$U_i = \prod_{j=1}^{m} (z_{ij})^{w_j}, \quad i = 1, ..., n \tag{17}$$

where z_{ij} are the normalized matrix elements and can be obtained by Eq. 2.

In Table 2, we presented RR average ratios (Type 1-5) according to each weighting methods in WPM. The comma-separated data in the table are ordered from the lowest to the highest number of alternatives. For example, the 1st and 5th data points are the RR percentages obtained with the low (n=4) and high (n=20) number of alternatives, respectively.

Accordingly first data points present the RR results for the decision problems which the numbers of alternatives are considerably low. When the number of alternatives is low, as the number of criteria increases, the RR type that increases the most is Type 3, while Type 2 and Type 5 are the least observed ones in general.

In second data points, the RR results for the decision problems which the numbers of alternatives are considerable in the lower-middle category are provided. When the number of alternatives is lower-middle and the number of criteria is low the highest RR occurrences are observed in Type 1, while with the increase in the number of criteria, the Type 3 has become be observed more. The least observed types of RR are actualized as Type 2 and 5.

Third data points present results for the decision problems with number of alternatives are considerable in the middle category. While Type 1, 3, and 4 have high occurrences ratios even with low number of criteria, they all goes up to higher levels with the increase in number of criteria with the exception of GSD which is RR free at least half of examples. Contrastingly, the Type 2 and 5 which can be observed similar to each other and lesser than others are not ascended rapidly with the increase in the number of criteria and remain lesser than 8%.

In forth data points, we present the results with number of alternatives are considerable in the upper-middle category for WPM. While the rate of observation of Types 1, 3 and 4 was at least 1 out of 3 even for a low number of criteria, this rate became upper than 50% when the number of criteria increased with the exception of RE for Type 3. The rate of increase in the incidence of Types 2 and 5, when the number of criteria increased, slightly increased.

Fifth data points present results for the decision problems with the maximum number of alternatives for WPM. While the rate of observation of Types 1, 3 and 4 were nearly at least 1 out of 2 even for a low number of criteria, this rate became upper than 60 % when the number of criteria increased and again with the exception for RE in Type 3. Contrastingly, Types 2 and 5 are not observed more than 6.2 % even when the number of criteria is at the highest.

In general, The GSD yielded better results than any of the other methods in every of RR types with the exception of Type 3 in which RE outperformed all. However, GSD in decision problems with low number of criteria possess more resistance to RR including the Type 3 than RE. The even the advantage of RE in Type 3 which is previously mentioned in the paper do not provide sufficient edge over GSD. Another general inference is that as the number of alternatives increases, the occurrences of Type 1, 3 and 4 increase, while Type 2 and 5 decrease.

Table 2. RR ratios of each entropy based methods in WPM

		SE	GSD	YE	RE
m=4	T1	23,47,60,71,76	9,23,34,41,49	19,42,56,65,71	25,41,59,69,75
	T2	9,7,5,4,3	2,2,2,1,1	6,5,3,2,1	10,4,3,2,1
	T3	21,43,62,72,75	11,24,33,42,49	17,40,55,65,73	20,27,33,42,47
	T4	15,44,58,70,76	6,18,29,42,46	13,36,51,64,71	16,38,53,66,73
	T5	8,8,5,3,4	3,1,1,1,0	6,6,3,1,3	9,4,3,4,2
m=8	T1	26,55,74,82,86	14,29,40,49,56	22,49,63,75,81	33,55,70,77,84
	T2	10,7,6,5,3	4,2,2,1,1	6,4,3,2,3	10,5,5,3,3
	T3	33,60,77,85,89	20,35,46,53,59	30,51,68,73,84	21,27,35,40,42
	T4	19,50,71,79,88	9,24,38,46,57	15,40,61,74,82	22,49,67,76,83
	T5	10,7,6,4,4	4,3,2,1,1	7,4,4,3,3	10,6,6,3,3
m=12	T1	30,61,77,85,92	15,33,45,54,63	24,53,65,80,87	34,58,73,83,89
	T2	10,8,5,5,5	4,3,2,1,2	8,6,5,4,2	12,8,4,4,3
	T3	40,67,80,91,93	22,40,51,61,67	32,60,74,81,86	22,25,31,38,41
	T4	17,53,74,85,90	10,25,38,51,59	14,44,63,77,84	23,53,69,78,88
	T5	9,8,6,7,5	5,2,2,1,1	8,6,5,3,3	11,8,4,4,4
m=16	T1	30,61,80,87,94	14,33,46,59,63	23,50,66,77,88	36,62,77,84,90
	T2	8,8,7,5,5	3,3,2,2,1	6,7,5,4,3	12,8,5,4,4
	T3	48,72,86,91,95	27,40,52,62,71	39,60,74,85,90	21,23,32,35,39
	T4	19,58,76,88,92	9,26,43,55,64	14,44,65,76,87	26,56,72,84,86
	T5	10,9,7,6,5	4,3,2,2,2	6,6,4,4,3	13,8,7,4,4
m=20	T1	32,65,80,91,93	15,33,45,57,60	27,54,67,80,88	38,68,77,86,90
	T2	11,9,7,5,5	5,3,2,2,2	8,6,4,3,3	14,9,7,4,3
	T3	52,75,87,94,95	31,44,55,63,70	44,65,79,85,90	20,27,30,33,36
	T4	20,54,76,88,93	12,28,42,56,63	17,48,65,76,87	27,61,78,86,88
	T5	11,9,7,5,6	6,2,3,2,0	9,6,4,4,3	13,10,7,4,3

We also evaluated the averages for each type of RR and overall average RR according to weighting methods. The results are provided in Figure 3. In WPM, while the observation rate of Type 2 and 5 are around 5%, the occurrences of Type 1, 3 and 4 are medium as about 50 %. According to results provided in Table 2 and also overall averages, the GSD is the most robust entropy based weighting scheme for WPM, while SE is the least. Figure 4 ranks the entropy functions in terms of resistance to RR in WPM. In these rankings made for different types of RRs with alternatives and criteria of different sizes, for example A8T2K4 indicates that the ranking is made for type 2 RR with 8 alternatives and 4 criteria.

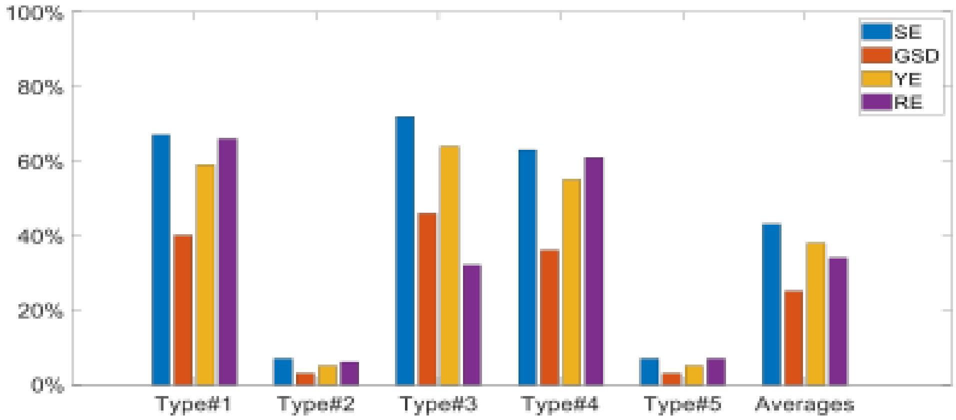


Figure 3. RR averages according to each weighting method in WPM

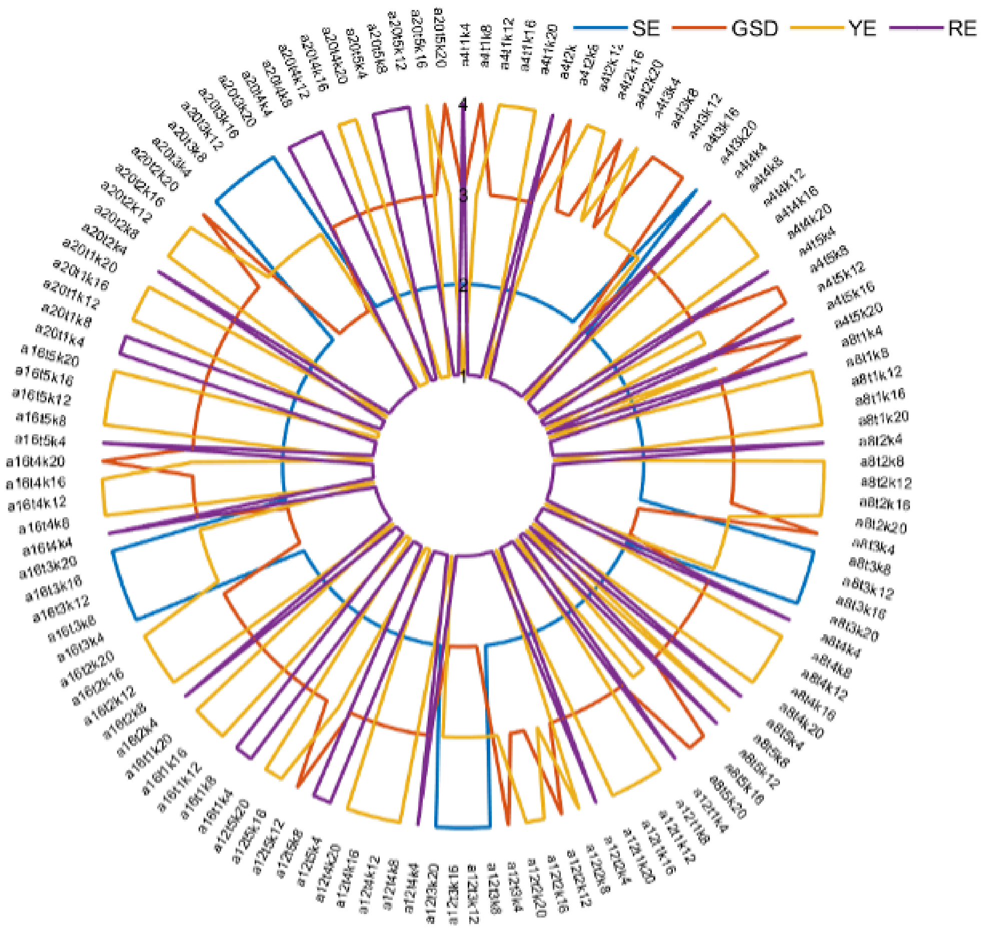


Figure 4. RR resistance ranks of weighting methods in WPM

3.3. Comparing entropy-based criteria weighting methods on criteria rank preservation and acceptancy

The sensitivity analysis concerning criteria importance levels is a common practice in MCDM (Chen et al., 2010). Since the criteria weights have direct effect on alternative ranking, their direct rank order stability is also another topic to focus on. Basically, if their rank preservation cannot maintain with adding or removing an alternative then a RR problem arouses. Additionally, Zavadskas & Podvezko (2016) and Ecer & Pamucar (2022), who proposed two objective weighting methods, IDOCRIW and LOPCOW, respectively, mentioned that the ratio of maximum to minimum criteria weights should be low enough to circumvent of dominance by a single criterion on decision problem which is not a sensible and acceptable case in practical. We also compared entropy-based methods in this acceptancy sense.

In Table 3, we presented RR percentages (Type 6-8) and weighting ranges with max/min ratio (RA) according to each weighting methods for 25000 decision problems. The comma-separated data in the table are ordered from the lowest to the highest number of alternatives. For example, the 1st and 5th data points are the RR percentages obtained with the low (n=4) and high (n=20) number of alternatives, respectively. In general (See Table 3), The SE yielded better results than any of the other methods in all RR types, albeit the differences may be negligible except in the case of the low number of criteria. However, the ratio values between the maximum and minimum criteria of SE seem to be quite large and problematic.

Table 3. RR and max/min ratios of each entropy based methods in criteria ranks

		SE	GSD	YE	RE
m=4	T6	54.9,44.9,44.9,33,29.8	58.8,47.1,47.1,33.5,32.5	62.9,48.8,48.8,36.6,34.1	73.2,67.2,67.2,60.9,58.5
	T7	33.7,33.5,33.5,29.3,30.2	39.5,37.3,37.3,30.9,30.3	45.6,41.8,41.8,36.3,33	58.7,42,42,32.8,30.6
	T8	62.9,45.5,45.5,33.2,31	66.2,50.6,50.6,36.7,34.4	69.3,51.1,51.1,39.4,33.9	78.1,68.9,68.9,59.9,59
	RA	20.5,3.9,3.9,2.3,2.1	1.4,1.3,1.3,1.2,1.2	3.4,2,2,1.6,1.5	1.7,1.4,1.4,1.2,1.2
m=8	T6	95.2,88.9,88.9,80.7,78.4	97.7,93.3,93.3,83.2,80.7	97.9,93.6,93.6,85,84.6	99,99.5,99.5,99.2,99.1
	T7	82.1,80.7,80.7,76.4,74.5	86.8,86.3,86.3,79.3,78.4	91.4,89.6,89.6,82.5,82.4	94.6,85.1,85.1,74.8,69.8
	T8	97.3,91.3,91.3,81.4,79.4	98.2,93.6,93.6,85.1,84.3	99.1,94.2,94.2,87.8,85.9	99.2,99.1,99.1,99.5,98.9
	RA	46.1,6.3,6.3,3.1,2.7	1.7,1.4,1.4,1.3,1.3	5.3,2.6,2.6,1.9,1.7	1.8,1.5,1.5,1.3,1.2
m=12	T6	99.8,99,99,95.7,94.7	100,99.4,99.4,98,98	100,99.7,99.7,98.7,98.5	100100100100100
	T7	97.1,96.5,96.5,96.2,94.1	98.3,97.6,97.6,97,95.4	99.6,99.4,99.4,98.1,97.5	99.9,96.2,96.2,94.2,90.3
	T8	99.9,99.2,99.2,97.3,96	100,99.6,99.6,97.9,97.3	100,99.8,99.8,99.2,98.2	100,100,100,100,100
	RA	64.9,8.3,8.3,3.7,3.1	1.8,1.5,1.5,1.3,1.3	6.4,3,3,2,1.9	1.9,1.5,1.5,1.3,1.2
m=16	T6	100,100,100,99.9,99.7	100,100,100,100,99.9	100,100,100,100,99.7	100100100100100
	T7	99.7,99.6,99.6,98.8,98.9	99.8,99.8,99.8,99.8,99.7	100,100,100,99.8,99.9	100,99.9,99.9,98.4,99
	T8	100,100,100,99.6,99.6	100,100,100,99.8,99.8	100,100,100,100,100	100100100100100
	RA	88.3,10.1,10.1,4.1,3.4	1.8,1.6,1.6,1.4,1.3	7.1,3.2,3.2,2.2,2	1.9,1.5,1.5,1.3,1.2
m=20	T6	100,100,100,100,99.8	100,100,100,100,100	100,100,100,100,100	100100100100100
	T7	100,100,100,99.9,100	100,100,100,100,100	100,100,100,100,100	100,99.9,99.9,100,99.7
	T8	100,100,100,100,100	100,100,100,100,100	100,100,100,100,100	100100100100100
	RA	213.2,10.7,10.7,4.5,3.7	1.9,1.6,1.6,1.4,1.4	8.5,3.4,3.4,2.3,2.1	1.9,1.5,1.5,1.3,1.2

The RR occurrence rates are rapidly growing as the number of criteria increase, aside from that the rank preservation is getting more complex with small number of alternatives. Accordingly, the large number of alternatives can be seen as a factor that increases the rank stability of the criteria. While SE benefits the most in terms of acceptability from the increase in the number of alternatives, it also suffers the most from the increase in the number of criteria.

We evaluated the averages for Type 6, 7, and 8 and also overall average RR according to weighting methods. The results are provided in Figure 5. The occurrences for all types and methods are higher than 80%, which might be seen a major concern for these methods. According to results provided in Table 3 and also overall averages, the SE is the most robust entropy based weighting scheme for criteria rank preservation, while RE is the least, albeit the differences are minor.

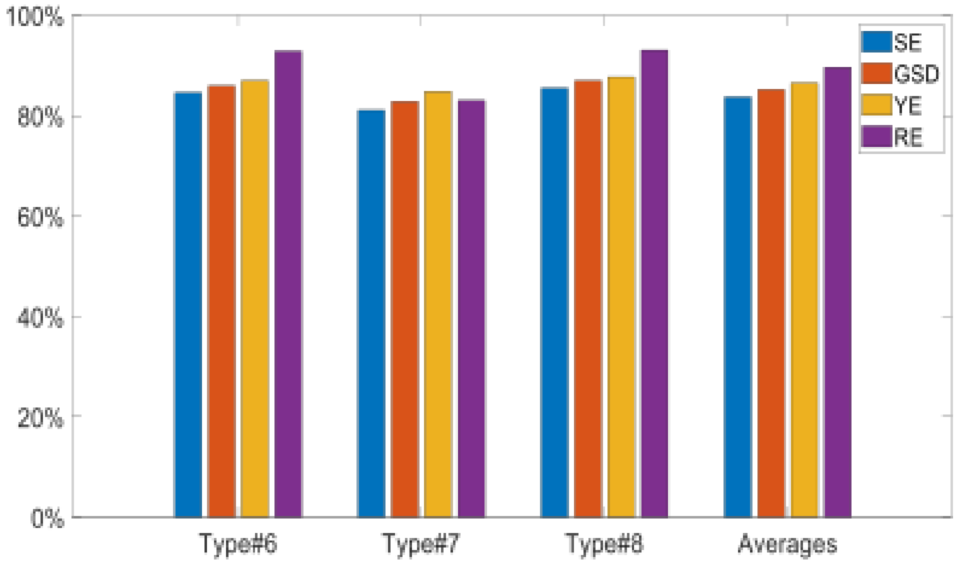


Figure 5. RR averages according to each weighting method in criteria ranking

We also examined the averages of minimum, maximum, and ratios (max/min) of criteria weights for each method. The results are provided in Table 4. According to results, the RE is come to the fore in acceptancy, while GSD following it tightly. Conversely, the SE has the least acceptable outcome, by far.

Table 4. Averages of minimum, maximum, and ratios for each method

	SE	GSD	YE	RE
$\min_j$	0.0523	0.0993	0.0742	0.0958
$\max_j$	0.198	0.1359	0.1592	0.1308
$\frac{\max_j}{\min_j}$	21.1135	1.4435	2.9991	1.4248

In general, WPM and WSM become more resistant to RR with GSD, while for WPM in particular, this difference is more obvious. SE performed poorly in RR (Type 1 to 5) for alternative ranking with WSM and WPM. The differences of RR rates (Type 6 to 8) for weighting methods in terms of criteria weight ranks are very small and negligible. However, while SE performed at max/min ratios, others can produce more suitable results. GSD approach provides, inter alia (RR), a very high acceptancy (less max/min ratios) as well.

We evaluated Spearman rank correlation between each method in terms of the ratio of maximum to minimum criteria weights to investigate the dispersion correlations of each method. We provided correlation results in Table 5 and accordingly RE is completely differentiated from the others, SE and YE have a very close correlation, while GSD differentiates slightly from the last two.

Table 5. Spearman Rank Correlation of each method based on dispersion

	SE	GSD	YE	RE
SE	1.0000	0.8739	0.9624	0.6087
GSD	0.8739	1.0000	0.8857	0.5863
YE	0.9624	0.8857	1.0000	0.5961
RE	0.6087	0.5863	0.5961	1.0000
Averages	0.86125	0.836475	0.86105	0.697775

4. An illustrative case study to investigate the entropy-based methods

To explore the dispersion characterization of each entropy-based method in more depth, we defined an illustrative problem with six criteria (C1,…,C6) which all of these criteria have different dispersion character on five alternatives (A1,…,A5).

Table 6. An Illustrated Example Data

	C1	C2	C3	C4	C5	C6
A1	900	1	1	4	550	998
A2	9	990	3	8	400	998
A3	1	980	990	24	960	998
A4	1	990	970	36	990	999
A5	9	975	980	16	500	998

In Table 6, C1 highlights only one alternative, while C2 highlights every observation point except skipping one. In C3, while the criterion dispersed observation points at extremes, they are not stand-alone. C4 provides less differentiation compared to the first three criteria. This situation continues to increase in C5 and C6, respectively. We provided the criteria weights according to each method for the case study in Table 7. This illustrative case study demonstrates the hypersensitivity of SE to the distribution of performance scores of alternatives in the criteria which is also previously mentioned in the literature (Mukhametzyanov, 2021). While YE suffered from same issue with SE, the RE completely failed in dispersion. While the GSD could reflect the contrast intensity, it was the only approach that was acceptable and not hypersensitive, simultaneously.

In Table 8, we presented the WSM and WPM rankings according to each weighting method, respectively. The WSM and WPM rankings with RE weights are completely different from others and inadmissible in the meantime. Apart from RE, while the three weighting methods yielded same ranking with WSM, they have differences in WPM rankings. However, when we evaluated the WSM and WPM rankings of the three weighting approaches within themselves, GSD has a magnitude difference of 4 in terms of rank difference magnitudes, while the others have a difference of 6 each. Accordingly, the WSM and WPM rankings of GSD seem more consistent.

Table 7. Criteria Weights

	C1	C2	C3	C4	C5	C6	C_{max}/C_{min}
SE	0.5952	0.0888	0.2012	0.0887	0.026	0	Inf
GSD	0.4253	0.111	0.1477	0.1267	0.1005	0.0889	4.8
YE	0.4248	0.109	0.2176	0.1538	0.0947	0.0001	4248
RE	0.037	0.1093	0.0254	0.1463	0.3174	0.3647	14.4

Table 8. WSM and WPM Ranking of Alternatives

	WSM							
	SE score	rank	GSD score	rank	YE score	rank	RE score	rank
A1	0.5906	1	0.4559	1	0.438	1	0.1671	4
A2	0.0395	5	0.0733	5	0.0569	5	0.1514	5
A3	0.122	3	0.1585	3	0.1695	3	0.2382	2
A4	0.1332	2	0.1759	2	0.1901	2	0.2611	1
A5	0.1148	4	0.1364	4	0.1455	4	0.1821	3
	WPM							
	SE score	rank	GSD score	rank	YE score	rank	RE score	rank
A1	0.0688	1	0.0593	1	0.037	2	0.0655	5
A2	0.0108	4	0.0224	5	0.0152	5	0.1208	4
A3	0.0105	5	0.0259	4	0.0271	4	0.1998	2
A4	0.0109	3	0.0273	3	0.0289	3	0.2143	1
A5	0.0369	2	0.0586	2	0.0608	1	0.1659	3

5. Conclusion

This study presents the use of different types of entropy and diversity measures in MCDM, particularly in criteria weighting. While SE is a very popular diversity metric in MCDM, GSD, YE and RE are metrics that have found their place in limited areas such as portfolio optimization. In our study to highlight these metrics in MCDM, these approaches were evaluated through RR, which is an important phenomenon in decision analysis. For this reason, we tested the synergy created by the ranking methods such as WSM and WPM and these weighting approaches together with a comprehensive simulation study over RR, and also examined the order change of the criteria importance levels in RR occurrences. We presumed that every objective criteria weighting methods based on decision matrix are susceptible of RR in some degree. However, it was interesting to find that SE, a widely used method in MCDM, underperformed than the other three approaches in RR. In comparing these entropy-based methods in terms of RR, the GSD come to the fore and outperformed all others in both WSM and WPM rankings.

After analyzing RR occurrences in each method with different number of criteria and alternative sets, we additionally interested in comparing these methods according to their range of weight vectors. While GSD and RE are the methods that differ positively from others in this respect, YE and particularly SE produced less acceptable weightings.

We also examined whether these methods cause similar decomposition in the same decision matrix with Spearman rank correlation and found the others to be similar except RE. After the examination how these methods produced contrast intensity through an illustrative example in detail, we found out the dispersion characteristics of RE is not working properly. Apart from this, both SE and YE suffered from the hypersensitivity which causes unacceptable results. In the meantime, the GSD performed better than these two approaches both in producing more acceptable weightings (less hypersensitive) and yielding more consistent rankings between WSM and WPM.

As a result, the GSD approach emerged as a robust solution to our research questions, demonstrating resistance to rank reversal, stability across different MCDM methods such as WSM and WPM, and resilience to hypersensitivity.

We also compared the results with other studies. Before presenting the comparison results with studies in the literature, it is important to emphasize that we used sum normalization in all methods and did not make any additional changes or procedures to prevent RR. In this context, we compared the results with the standard sum normalization TOPSIS in Farias Aires & Ferreira (2019) and with Baykasoğlu & Ercan (2021) standard sum normalization WASPAS for types 2 and 5 of RR. The RR results obtained with GSD-WSM are better than TOPSIS and similar to WASPAS. However, the results obtained with GSD-WPM, on the other hand, are better in terms of RR than both TOPSIS and WASPAS.

Accordingly, the main contributions of this study are the following: (I) the defining of different types of entropy and diversity indexes in criteria weighting; (II) the examining and comparing of objective criteria weighting methods through RR analysis, considering entropy-based approaches; (III) generating a framework for comparing criteria weightings in terms of RR; (IV) the exploring dispersion characteristics of entropy-based approaches with an illustrative case in detail; (V) the proposing of a proper alternative to SE as GSD which is more persistent to RR, less sensitive to distribution of domain and outputs more acceptable weightings.

As a result of our analysis, practitioners, managers and decision makers may prefer more robust entropy functions (e.g. GSD) than popular (e.g. SE) to prevent possible RR when weighting criteria, especially in cases where they are undecided about the relevance of some alternatives to the selection problem. In this way, the reliability and validity of the results will increase. Also they can acquire more acceptable weights with GSD than standard SE. The theoretical contribution of the study to the field of MCDM is that it presents the comparison framework of criteria weighting methods in terms of RR and new entropy functions to the methodology.

The limitations of the study are that only WPM and WSM were used in terms of ranking methods and the proposed method was not used in a real-life problem. Future works can consider the GSD approach to assign criteria weights in various real-world decision problems and cooperated with different types of ranking methods. Furthermore, there is significant amount of room in future studies for developing criteria weighting methods that would be immune to RR. To achieve this, the key point may be the normalization procedure, as an insight.

Declarations

Conflict of interest The author(s) have no competing interests to declare that are relevant to the content of this article.

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