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A robust optimization approach to address correlation uncertainty in stock keeping unit assignment in warehouses

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Abstract

This study addresses the problem of assigning correlated Stock Keeping Units (SKUs) to storage locations under uncertain SKU correlation conditions. The objective is to allocate SKUs within the forward picking area of a warehouse to minimize the total picking distance. The correlation of SKUs in demand patterns is considered while assigning the SKUs to storage locations. The correlation among the SKUs is identified based on the joint distribution concept. We formulate the problem as a Quadratic Assignment Problem (QAP), which becomes computationally intractable at large scales due to its complexity. The QAP model is linearized to mitigate this challenge, and a robust counterpart is developed to handle uncertainty effectively. The robust model was evaluated through various small-scale scenarios. While it yielded optimal results within an efficient time frame for small-scale problems, the solution time increased significantly as the problem size expanded.

Keywords SKUs assignment problem, Robust optimization, Demand correlation, Mathematical modeling

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1. Introduction

Warehouse management takes a critical role in the supply chain, particularly with the widespread adoption of e-commerce and the globalization of production. Many manufacturers and e-commerce companies utilize warehouses to distribute goods efficiently and cost-effectively. Given the vast number of products handled in a warehouse, the costs associated with receiving, storing, and shipping can be substantial. Warehouse Management Systems (WMS) are designed to optimize operations by reducing the highest cost components. According to Bartholdi et al. (2008), approximately 65% of warehouse costs are attributed to the picking process. Similarly, Tompkins et al. (2010) reported that order picking accounts for 55% of total warehouse operating expenses, while the remaining costs are associated with shipping, storage, and receiving. Since travel time or distance in the picking process is a key performance indicator, minimizing it is a primary objective. Consequently, focusing on optimizing the picking process can mitigate the overall operational costs of warehouse management.

The placement and retrieval of related products have been addressed to some extent in the literature. The process of storing and retrieving products based on customer orders can be highly time-consuming. When products are placed randomly on warehouse shelves, retrieval times increase significantly. Often, when customers place orders, there is a high likelihood that they will simultaneously order related products. Consequently, arranging related products near one another within the warehouse can substantially reduce retrieval times. For instance, a customer ordering a toothbrush is likely to also order toothpaste. If two related products are stored in close proximity within the warehouse, the time it takes for a picker to retrieve these products can be minimized. Randomly placed products increase the time it takes to pick products from the warehouse, thus making warehouse management operations inefficient.

This study examines the slotting of correlated SKUs, where certain SKUs are typically ordered together. Consequently, when assigning SKUs to available storage locations, it is essential to account for their affinity. Placing correlated SKUs in close proximity to one another can reduce the picking distance. In other words, by incorporating product affinity, the operational costs of the warehouse can be significantly minimized. To this end, we consider various numbers of racks within the forward picking area. It is assumed that the racks have an equal number of SKUs to be assigned, as illustrated in Figure 1. The problem is modeled with a framework of the quadratic assignment problem, and it becomes computationally intractable as the number of SKUs and racks increases. With an uncertain demand condition, a robust modeling approach is proposed for assigning a specific number of SKUs to storage locations, considering the uncertainty in incoming orders. The correlation data between SKUs is determined based on the concept of joint distribution. The upper and lower bounds of the nominal value are derived from the literature and calculated based on expected values to in robust counterpart of model.

The rest of the paper is structured as follows: Section 2 presents a review of the literature on SKU assignments and robust optimization problems. Section 3 presents the details of the quadratic, linear, and robust modeling approaches, along with the methodology for calculating SKU correlations. Section 4 provides information on the solution of the model and the analysis of the results. Finally, Section 5 discusses the conclusions and directions for future work.

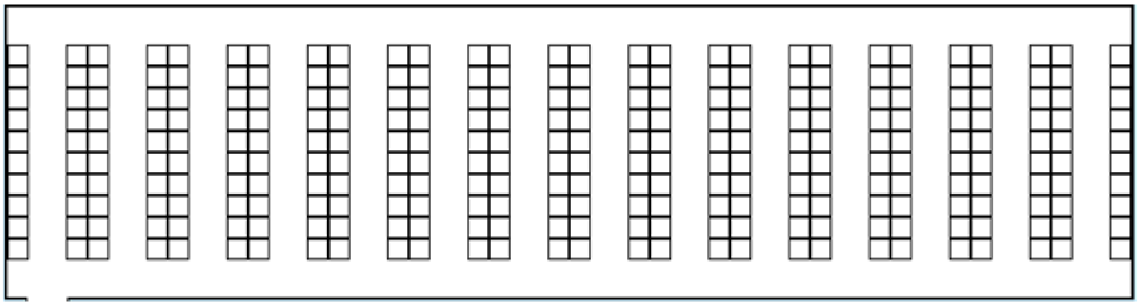

Figure 1. An example of layout in forward picking area

2. Literature Review

The task of assigning SKUs to storage locations is commonly referred to as the slotting problem. The studies, including slotting, have been searched in the journal databases. A significant number of papers have investigated the slotting problem. Kim & Smith (2012) studied the assignment of SKUs to slots in zone-based carton picking distribution systems, where the items to be picked had already been determined. They proposed a simulated annealing heuristic approach to solve the slotting problem due to the characteristic of being large-scale nature. Mantel et al. (2007) proposed a new strategy to assign SKUS to storage locations in a warehouse. In their study, an assignment model has been presented by considering the correlation between the products ordered together, instead of the number of orders placed, called the Cupe per Order Index (COI). Xiao & Zheng (2012) designed a system of order picking to minimize zone visits in storage areas by considering storing items with demand dependencies. They proposed a mathematical model and two heuristic algorithms to assign items in the storage area. Islam & Uddin (2023) conducted a detailed literature review about correlated SKUS assignment problems. The study evaluated correlated slot assignment problems and problem-solving approaches. (Ma et al., 2022) introduced a mixed integer programming model for commodity storage assignment problems, aiming to assign products to appropriate locations while taking customer demand patterns into account. The authors constructed a variable neighborhood search and simulated annealing search-based framework to test small, medium, and large instances to compare with the state-of-the-art methods. Zhou et al. (2014) investigated the impact of demand correlation among SKUS on the picking process to minimize total picking time. The study proposed a particle swarm algorithm with a cube-per-order index-based initial solution to solve the assignment problem within a reasonable time.

The quadratic assignment problem (QAP) framework has been widely applied in various studies. For instance, Elshafei (1977) employed QAP to optimize hospital layouts by analyzing existing layouts with predefined locations and assigning an equal number of departments to these locations. In the context of warehouse optimization, this study focuses on the assignment of stock-keeping units (SKUs) to racks in the forward picking area. The SKU assignment problem is classified as a combinatorial optimization problem, which is known for its computational complexity and difficulty in achieving optimal solutions within a reasonable time frame. To address general QAP instances, Ahuja et al. (2000) developed a greedy genetic algorithm. From the perspective of warehouse picking operations, Hsu et al. (2005) proposed a genetic algorithm-based order batching approach to enhance efficiency across different batch structures and warehouse layouts. Furthermore, Poulos et al. (2001) applied a Pareto-optimal genetic algorithm to the warehouse replenishment problem, aiming to optimize resource utilization, minimize costs, and improve customer service.

In recent years, the Robust Optimization (RO) approach has been extensively utilized in many studies to model uncertainty in data. Bertsimas & Sim (2003) developed the robust counterpart of a linear model, significantly reducing the solution time. Bertsimas & Sim (2004) also formulated the robust counterpart of a MILP model, facilitating more efficient solutions for optimization problems with integer variables. The robust optimization method has been utilized for various problems in many fields. Remodeling of problems with uncertainty parameters allows the problem to be solved in a shorter time. Dundar et al. (2019) and Dundar et al. (2022) utilized the RO modeling approach to address the uncertainty in renewable energy planning studies. Costello et al. (2021) and Dundar et al. (2017) developed the robust counterpart of linear models that aimed to solve local food problems. The Robust Optimization (RO) approach enables decision-makers to systematically manage risk scenarios by incorporating uncertainty into the decision-making process, thereby enhancing solution reliability and robustness. In this study, the uncertainty in the correlation between SKUs is modeled using the RO approach. To the best of our knowledge, this study distinguishes itself from the existing literature by employing the joint distribution approach to model the correlation between SKUs as an uncertain parameter and addressing the problem using the robust optimization approach.

3. Problem Formulations

A Quadratic Assignment Problem(QAP) approach is proposed to optimally assign the correlated SKUs in the forward picking area as carton unit-loads to the storage location. The mathematical formulation of the problem is presented below. The objective function (Eq. 1) aims to minimize the total distance among the correlated SKUs, thereby reducing the picker's travel distance when fulfilling orders. Let θ denote the set of correlated SKUs, and $\mathfrak{L}$ is the set of storage locations. Let $c_{ij} > 0$ be the correlation of product i and j ordered together. Moreover, let $d_{kr} > 0$ be distance between storage location k and r.

$$\psi = \min \sum_{i,j \in \theta} \sum_{k,r \in \mathcal{L}} c_{ij} d_{kr} \chi_{ik} \chi_{jr} \tag{1}$$

The constraint (Eq. 2) restricts that each product i can be assigned only one storage location k. Similarly, the constraint (Eq. 3) shows that each location k can be assigned to only one product i.

$$\sum_{k \in \mathcal{L}} \chi_{ik} = 1, \forall i \in \theta \tag{2}$$

$$\sum_{i \in \theta} \chi_{ik} = 1, \forall k \in \mathcal{L} \tag{3}$$

The following assumptions are considered in the development of the mathematical model: (1) each storage location is assigned to a single SKU, (2) the storage system utilizes double-deep racks, and (3) the distance between two storage locations is assumed to be equal, measured from the center of one storage location to the center of another.

3.1. Linearization of QAP (LQAP) model

The objective function of the QAP model includes nonlinear terms. Linearizing these terms facilitates the formulation of a robust counterpart of the model and ensures its solvability within a reasonable time frame. The linearized version of the QAP(LQAP) model is presented below. A new variable γ_{ijkl} is introduced, which enforces the relationship between $\gamma_{ijkl} = \chi_{ik} \cdot \chi_{jl}$

$$
\begin{aligned}
\min \quad & \sum_{\{(ij)\in\theta \ | \ i<j\}} \sum_{k,r\in\mathcal{L}} |c_{ij}| d_{kr} \gamma_{ijkr}, \\
\text{s.t.} \quad & \sum_{k\in\mathcal{L}} \chi_{ik} = 1, \forall i \in \theta, \\
& \sum_{i\in\theta} \chi_{ik} = 1, \forall k \in \mathcal{L}, \\
& \gamma_{ijkr} \leq \chi_{ik}, \quad \forall (i,j) \in \theta | i < j, \forall (k,r) \in \mathcal{L} \\
& \gamma_{ijkr} \leq \chi_{jr}, \quad \forall (i,j) \in \theta | i < j, \forall (k,r) \in \mathcal{L}, \\
& \chi_{ik} + \chi_{jr} - 1 \leq \gamma_{ijkr}, \quad \forall (i,j) \in \theta | i < j, \forall (k,r) \in \mathcal{L} \\
& \chi_{ik} \in \{0,1\}, \quad \gamma_{ijkr} \in \{0,1\}, \quad \forall i,j,k,r
\end{aligned}
\tag{4}
$$

3.2. Robust counterpart of LQAP model

The LQAP model, as previously described, represents a combinatorial optimization problem. Bertsimas & Sim (2003) proposed a robust counterpart for the objective function incorporating a budget constraint Γ_0. The uncertain product frequency parameter, $\tilde{c}_{ij}$, $i, j \in \theta$ assumed to lie within the interval $[c_{ij}, c_{ij} + \delta_{ij}]$. The aim is to determine a feasible solution γ_{ijkr} that minimizes the maximum deviations in frequency parameters while ensuring that at most Γ_0 of the parameters, $\tilde{c}_{ij}$ are allowed to deviate.

$$
\sum_{\{(i,j)\in\theta | i<j\}} \sum_{(k,r)\in\mathcal{L} | k\neq r} c_{ij} d_{kr} \gamma_{ijkr} + z_0 \Gamma_0 + \sum_{(i,j)\in\tilde{\theta}} \tilde{v}_{ij} \tag{5}
$$

The final two terms in the objective function, which ensure robustness, are based on the Robust Optimization (RO) framework introduced by Bertsimas & Sim (2003) to address input data uncertainty.In this framework, the objective function coefficients subject to uncertainty represent the frequency of orders among stock-keeping units (SKUs). The uncertain frequency is denoted as c_{ij}, here this parameter is assumed to be bounded and symmetrically distributed within the interval $[|(c)_{ij} - \hat{c}_{ij}, |(c)_{ij} + \hat{c}_{ij}]$. Here, $|(c)_{ij}$ denotes the nominal frequency value, while $\hat{c}_{ij}$ represents the maximum allowable deviation from this nominal value. Additionally, it is assumed that all uncertain parameters fluctuate independently. Let $\tilde{\theta} = \{(i,j) \mid \hat{c}_{ij} > 0\}$, denote the subset of products whose frequency parameters may deviate from their nominal values. Bertsimas & Sim (2004) introduced the robustness control parameter Γ_0, which takes an integer value and allows users to adjust the level of robustness in the objective function. When $\Gamma_0 = 0$, all parameters remain fixed at their nominal values, leading to a deterministic model without robustness. In the proposed model, when $\Gamma_0 = |\tilde{\theta}|$, all parameters within the subset $\tilde{\theta}$ are permitted to vary from their nominal values. Under these conditions, the optimal solutions are determined by considering the worst-case scenario, following the approach outlined in Soyster (1973). For Γ_0 values that fall between these two limits., the term $z\Gamma_0 + \sum_{(i,j)\in\tilde{\theta}} \tilde{v}_{ij}$ in the objective function quantifies the additional cost incurred due to the Γ_0 worst-case-impact values of the decision variables γ_{ijkl}. The constraints (Eq. 6 - Eq. 10) have been reformulated and added to the linearized model to control the uncertainty of parameters via Γ_0 in the objective function at the desired level.

$$z_0 + \tilde{\upsilon}_{ij} \geq \hat{\gamma}_{ij}\tilde{n}_{ij} \quad \forall (i,j) \in \tilde{\theta} \tag{6}$$

$$-\tilde{n}_{ij} \leq w_{ij} \leq \tilde{n}_{ij} \quad \forall (i,j) \in \theta \tag{7}$$

$$\tilde{\upsilon}_{ij} \geq 0 \quad \forall (i,j) \in \tilde{\theta} \tag{8}$$

$$\tilde{n}_{ij} \geq 0 \quad \forall (i,j) \in \theta \tag{9}$$

$$z_0 \geq 0 \tag{10}$$

3.3. Demand correlation of SKUs frequency

Changes in SKU demand were analyzed using joint and marginal distributions to determine the correlation of SKUs being ordered together. The correlation data was used as nominal data for the frequency parameters. Additionally, to apply robust optimization, it is crucial to establish the upper and lower bounds of the nominal parameters. The correlation matrix, which represents the frequency of SKUs in the picking process, along with the upper and lower bounds, is determined by following the steps below.

Step 1: Determining the joint distributions and marginal distributions of SKUs.

Let's assume that SKUs are given then the marginal distribution of each SKUs is denoted with $P(S_i)$ and is calculated in Eq. 11. In similar way, the marginal distribution of $S_1 = s_1, S_2 = s_2, ..., S_n = s_n$ n SKUs can be determined from joint probability distribution $P_{\{s_1,s_2,...,s_n\}}(S_1, S_2, ..., S_3)$.

$$P(S_i) = \sum_{s_1,s_2,...,s_n \neq s_i} P(S_1 = s_1, S_2 = s_2, ..., S_n = s_n) \tag{11}$$

Step 2: Estimating the covariance and correlation of SKUs

Based on the data from the joint probability density function the covariance of the SKUs is estimated as given in Eq. 12

$$\mathrm{Cov}(S_i, S_j) = \mathbb{E}[S_i S_j] - \mathbb{E}[S_i]\mathbb{E}[S_j] \tag{12}$$

In above equation $\mathbb{E}[S_i]$ is the expected value of SKUs i (S_i) and calculated based on $\mathbb{E}[S_i] = \sum_s s(S = s)$. $\mathbb{E}[S_i S_j]$ is the expected value of product of SKUs i and SKUs j, which is provided in Eq. 13

$$\mathbb{E}[S_i S_j] = \sum_{s_1,s_2,...,s_n} s_1 s_2 P(S_1 = s_1, S_2 = s_2, ..., S_n = s_n) \tag{13}$$

The correlation between two SKU i and SKU j is is estimated as given in Eq. 14

$$\rho(S_i, S_j) = \frac{\mathrm{Cov}(S_i S_j)}{\sigma_{S_i}\sigma_{S_j}} \tag{14}$$

where the standard deviation of SKUs i (σ_{S_i}) is $\sqrt{\mathbb{E}[S_i^2] - (\mathbb{E}[S_i])^2}$

Step 3: Creating covariance matrix to generate nominal values of SKUs demand frequency.

As aforementioned the nominal value of the correlated skus obtained by estimating the correlation value between SKUs. The correlation matrix for demand frequency is given below.

$$\mathbb{C} = \begin{pmatrix} 1 & \rho(S_1, S_2) & \cdots & \rho(S_1, S_n) \\ \rho(S_2, S_1) & 1 & \cdots & \rho(S_2, S_n) \\ \vdots & \vdots & \ddots & \vdots \\ \rho(S_n, S_1) & \rho(S_n, S_2) & \cdots & 1 \end{pmatrix} \tag{15}$$

Step 4: Determining the lower and upper bounds of the coefficient of the correlation values with the Fréchet-Hoeffding Bounds approach (Olvera Astivia et al., 2020).

The lower bound ($\rho_{\min}$) for each $\rho(S_i, S_j)$ is determined by using theoretical bounds identified in Eq. 16.

$$\rho_{min}(S_i, S_j) = \frac{\mathbb{E}[\min(S_i, S_j)] - \mathbb{E}[S_i]\mathbb{E}[S_j]}{\sigma_{S_i}\sigma_{S_j}} \tag{16}$$

where $\mathbb{E}[\min(S_i, S_j)]$ is s estimated based on the Eq. 17

$$\mathbb{E}[\min(S_i, S_j)] = \sum_{s_1}\sum_{s_2} \min(s_1, s_2) P(S_1 = s_1, S_2 = s_2) \tag{17}$$

The upper bound ($\rho_{\max}$) for each $\rho(S_i, S_j)$ is provided by using theoretical bounds identified in Eq. 18.

$$\rho_{\max}(S_i, S_j) = \frac{\mathbb{E}[\max(S_i, S_j)] - \mathbb{E}[S_i]\mathbb{E}[S_j)]}{\sigma_{S_i}\sigma_{S_j}} \tag{18}$$

where $\mathbb{E}[\max(S_i, S_j)]$ is s estimated based on on following equations.

$$\mathbb{E}[\max(S_i, S_j)] = \sum_{s_1}\sum_{s_2} \max(s_1, s_2) P(S_1 = s_1, S_2 = s_2) \tag{19}$$

A numerical example of the estimation of demand correlation is illustrated in Table 1.

Table 1. An illustration for the probability of two correlated SKUs with marginal distribution values

$P_{S_1,S_2}(s_1,s_2)$		s_2					$P_{S_2}(s_2)$
		0	1	2	3	4	
s_1	0	0.15	0.04	0.03	0.02	0.02	0.26
	1	0.02	0.09	0.07	0.05	0.06	0.29
	2	0.01	0.03	0.13	0.15	0.13	0.45
$P_{S_1}(s_1)$		0.18	0.16	0.23	0.22	0.21	

In the illustrated example, within any given order from the total incoming orders, a maximum of 2 units of SKU $S_1 \in \{0, 1, 2\}$ can be ordered. Similarly, a maximum of 4 units of SKU $S_2 \in \{0, 1, 2, 3, 4\}$ can be demanded. The table shows that, for instance, the probability of ordering 1 unit of S_1, and 2 units of S_2 together is $P_{S_1,S_2}(s_1, s_2) = 0.07$ and the probability of ordering 2 unit of S_1, and 2 units of S_2 is $P_{S_1,S_2}(2, 2) = 0.13$. The table also provides the marginal distribution for each SKUs(S) value. For example, while $P_{S_1}(0) = 0.26$, $P_{S_1}(2)$ takes a value of 0.45. Following the similar steps, the marginal distributions of S_2, P_{S_2}, have also been determined. For instance, while $P_{S_2}(0)$ = 0.18, $P_{S_2}(4)$ takes a value of 0.21.

After determining the marginal distributions of the SKUs, the covariance between the two SKUs can be calculated based on Eq. 12. To compute the covariance ($\mathrm{Cov}(S_1, S_2)$), the expected values of $\mathbb{E}[S_1]$ and $\mathbb{E}[S_2]$ must be determined along with the $\mathbb{E}[S_1 S_2]$. Regarding Table 1, one can find $\mathbb{E}[S_1] = 1.19$ and $\mathbb{E}[S_2] = 2.12$. Accordingly, $\mathbb{E}[S_1 S_2]$ is 3.14. Since the value of $\rho(S_1, S_2)$ depends on σ_{S_1} and σ_{S_2}, one can find the $\sigma_{S_1} = 0.821$ and $\sigma_{S_2} = 1.388$. The correlation between SKUS 1 and SKUS 2 can be estimated as $\rho(S_1, S_2) = 0.549$. Meaning that there is a moderate positive correlation between SKUS 1 and SKUS 2. Similarly, the pairwise correlation values, c_{ij}, among all SKUs are calculated, and the C matrix is constructed.

4. Computational Results

In this section, ROLQAP and the LQAP models for various scenarios are analyzed. The ROLQAP model was tested on a computer equipped with a 13th Gen Intel® Core™ i7-13700H processor and 64 GB of RAM, using Python 3.11 and Gurobi 12.0. As mentioned previously, for the computational test, it is assumed that the number of SKUs to be assigned is equal to the number of storage locations. To evaluate the results of the proposed models, 5, 10, and 12 SKUs were tested under the uncertainty of the correlation parameters of the SKUs and the deterministic scenarios. The distance between adjacent rack locations (d_{kr}) was assumed to be 2 meters. The correlation coefficient between SKUs was generated, $c_{ij} \sim \mathcal{U}\{-1, 1\}$, which aligns with the uniform distribution for experimental studies.

Table 2. ROLQAP model results for 5 SKUs and storage location

Γ_0	% of deviation	Objective function value	CPU time (sec)
	10	39.9	0.12
0	30	39.9	0.16
	50	39.9	0.17
	10	40.6	0.17
1	30	42.0	0.13
	50	43.4	0.14
	10	41.3	0.18
2	30	44.0	0.19
	50	46.7	0.17
	10	41.7	0.18
3	30	45.3	0.15
	50	48.9	0.18
	10	42.1	0.19
4	30	46.6	0.16
	50	51.0	0.15
	10	42.5	0.14
5	30	47.7	0.15
	50	52.9	0.15

As shown in Table 2, we ran the ROLQAP model for five SKUs and five storage locations. We analyzed the model under 10%, 30%, and 50% deviation scenarios by adjusting the upper and lower bounds of the nominal correlation value. If the deviation value is increased from %10 to %50, the objective function value shows a significant increase. This is due to the increase in the worst-case value that the nominal value can take. For example, in the $\Gamma_0 = 3$ scenario, the objective function value is 41.7 for a 10% deviation, whereas it rises to 48.9 when the nominal value deviates by 50%. As previously mentioned, we initialized the Γ_0 value at zero and gradually increased it by one, representing the number of SKU correlations that could deviate from their nominal values. As expected, the objective function value increased as the Γ_0 value grew. For instance, in the 10% deviation scenario, the objective function value was 40.6 when $\Gamma_0 = 1$, whereas it increased to 42.5 when $\Gamma_0 = 5$. As shown in Table 2, the average CPU time for five SKUs and five storage locations was calculated as 0.16 seconds. This result indicates that the ROLQAP model can achieve an optimal solution quite efficiently for the given number of SKUs.

Similarly, we ran the ROLQAP model for 10 SKUs and 10 storage locations under different Γ_0 and deviation scenarios. As shown in Table 3, for Γ_0, meaning there is no deviation in correlation, GUROBI achieves an optimal objective function value of 293.7 within an average of 70 seconds CPU time. We observe that as the Γ_0 value increases, the objective function value also increases. Specifically, for $\Gamma_0 = 0$ and a 10% deviation, the objective function value is 293.7, whereas for $\Gamma_0 = 10$ under the same deviation level, the objective function value rises to 307.1. It can be observed that when $\Gamma_0 = 10$, increasing the deviation from 10% to 50% results in a rise in the objective function value from 307.1 to 360.5.

Table 3. ROLQAP model results for 10 SKUs and storage location

Γ_0	% of deviation	Objective function value	CPU time (sec)
0	10	293.7	68.1
	30	293.7	69.5
	50	293.7	70.7
2	10	297.0	63.4
	30	303.5	75.1
	50	310.0	65.1
4	10	299.8	65.4
	30	312.0	69.5
	50	324.1	70.9
6	10	302.5	65.5
	30	319.9	70.5
	50	337.4	72.2
8	10	304.9	67.2
	30	327.2	69.2
	50	349.5	65.9
10	10	307.1	61.6
	30	333.8	71.8
	50	360.5	39.1

We analyzes the model for 12 SKUs and 12 storage locations to examine the impact of increasing SKUs and storage locations on ROLQAP as presented in Table 4. For $\Gamma_0 = 0$, and a 10% deviation, the ROLQAP model yields an objective value of 501.7. The average CPU time for $\Gamma_0 = 0$, is 2059 seconds. Increasing the nominal deviation value (c_{ij}) for any given Γ_0 leads to a rise in the objective value. For example, in the ROLQAP model, when $\Gamma_0 = 8$ with a 10% deviation, the objective value is 516.7, whereas for $\Gamma_0 = 8$ with a 50% deviation, this value increases to 575.3. By considering the same deviation conditions, an increase in the number of parameters that can deviate from the nominal value leads to an increase in the objective function value in the worst-case scenario. For example, with $\Gamma_0 = 0$ and a 50% deviation, the objective function value is 501.7, whereas for $\Gamma_0 = 12$, the objective value reaches 599.6. Furthermore, as a result of the analysis, it has been determined that the average CPU time is observed to be 1754 sec for any scenario.

Table 4. ROLQAP model results for 12 SKUs and storage location

Γ_0	% of deviation	Objective function value	CPU time (sec)
	10	501.7	2026
0	30	501.7	2038
	50	501.7	2113
	10	506.3	1225
2	30	515.6	1599
	50	524.7	2016
	10	510.4	3192
4	30	527.3	2118
	50	543.9	1328
	10	516.7	1142
8	30	545.8	1117
	50	575.3	1155
	10	519.1	1015
10	30	553.3	1632
	50	587.4	1027
	10	521.6	2046
12	30	560.6	1305
	50	599.6	1604

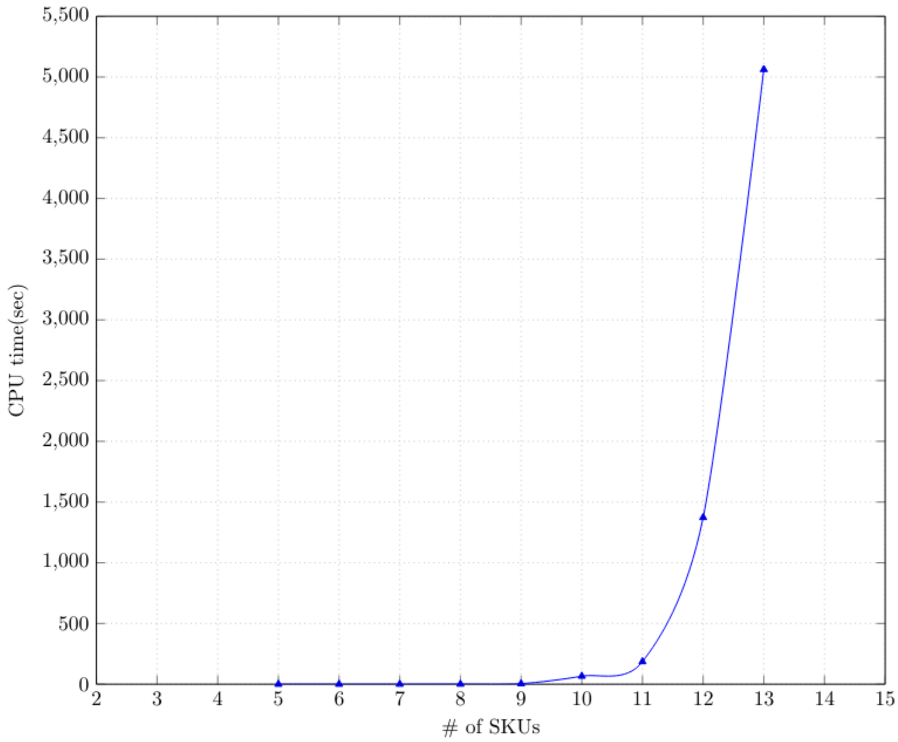

Figure 2. CPU time vs. Number of SKUs for $\Gamma_0 = 5$ and 30% deviation of nominal values

As the number of SKUs increases, the solution time of the ROLQAP model rises dramatically. The ROLQAP model was tested with different SKU numbers under the conditions of $\Gamma_0 = 5$ and 30% deviation. As shown in Figure 2, when the number of SKUs is between 5 and 10, an optimal solution can be obtained within a reasonable time frame. However, when the number of SKUs exceeds 10, a significant increase in solution time is observed. For instance, when the number of SKUs is 12, the solution time is 1372 seconds, whereas increasing the SKU count to 13 raises the CPU time to 5062 seconds.

5. Conclusion

In conclusion, we consider the uncertainty in the correlation of SKUS demand while assigning products to available storage locations. The main aim is to locate highly correlated SKUs and minimize the total travel distance of workers during the order-picking process. Marginal distributions and expected values are computed within the context of the joint distribution to quantify SKU correlations. The problem is modeled as a Quadratic Assignment Problem (QAP), and a linearization version is presented. The ROLQAP model's robust counterpart is created by considering the Bertsimas & Sim (2004) method to reduce the uncertainty model's computational burden. ROLQAP model is tested under the scenarios of various γ_0, deviation amount, and SKUs. It is remarked that with SKUs over 13, the problem size makes computational time intractable. For future studies, one can propose a heuristic or metaheuristic method to solve the large-scale problem of the correlated SKUs assignment.

Declarations

Conflict of interest The author(s) have no competing interests to declare that are relevant to the content of this article.

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