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Analysis of the Computational Performance in Traveling Salesman Problem: An Application of the Grey Prediction Hybrid Black Hole Algorithm

Mehmet Fatih Demiral [1]

[1] Faculty of Engineering-Architecture, Burdur Mehmet Akif Ersoy University, Burdur, Türkiye

Abstract

Grey prediction evolution algorithm (GPEA) is a nature-inspired intelligent approach applied to global optimization and engineering problems in 2020. The performance of the GPEA is evaluated on benchmark functions, global optimization, and tested on six engineering-constrained design problems. The comparison shows the effectiveness and superiority of the GPEA. Although the pure GPEA is better than other algorithms in global optimization, and engineering problems, it shows poor performance in combinatorial optimization. In this work, GPEA hybridizes with the black hole algorithm and tabu search for the event horizon condition. Besides, the grey prediction hybrid black hole algorithm (GPHBH) is implemented with heuristics, such as 2-opt, 3-opt, and k-opt swap, and tries to improve with constructive heuristics, such as NN (nearest neighbor), and k-NN. All the algorithms have been tested under appropriate parameters in this work. The traveling salesman problem has been used as a benchmark problem so eight benchmark OR-Library datasets are experimented with. The experimental solutions are presented as best, average solutions, standard deviation, and CPU time for all datasets. As a result, GPHBH and its derived forms give alternative and acceptable solutions to combinatorial optimization in admissible CPU time.

Keywords Grey Prediction Evolution Algorithm, Heuristics, Hybrid Black Hole Algorithm, Metaheuristics

Jel Codes C60, C61, C63

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Correspondence

M. F. Demiral
mfdemiral@mehmetakif.edu.tr

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1. Introduction

Modern technology brings new perspective to complex problems by putting forward novel metaheuristics, such as evolutionary algorithms (Holland, 1975; Koza, 1992), biology-inspired algorithms such as ant colony optimization (Peker et al., 2013), worm optimization (Arnaout, 2014), artificial bee colony optimization (Szeto et al., 2011), physics-inspired algorithms such as optics-inspired algorithm (Husseinzadeh Kashan, 2015), water-wave optimization (Zheng, 2015), black hole algorithm (Hatamlou, 2018), chemistry-inspired algorithms such chemical reaction optimization algorithms (Siddique & Adeli, 2017).

This article uses the grey prediction model (Deng, 1982; Tien, 2012) and proposes a new prediction hybrid black hole algorithm to put the traveling salesman application into the literature. Finding an optimal solution begins with an initialization operator to assign a trial population and randomly create a new generation. Instead of using crossover and mutation operators after the initialization, the grey prediction algorithm uses the reproduction and selection operator.

In the reproduction operator, *egm11 reproduction*, the alpha and beta model parameters of the even grey model are estimated. The new population improves with each iteration in the grey prediction algorithm. Firstly, raw data substitutes with objective function values, and then intermediate information is computed by the minimum operator evaluation. The prediction sequence of the transformed data is calculated by the transformed data sequence. The prediction sequence of the raw data is calculated. Finally, the most promising candidate solutions are selected by the selection operator.

The most frequently used grey prediction evolution algorithm is based on the even grey model (EGM (1,1)). Inspired by the EGM (1,1), the article proposes a new metaheuristic, named the GPHBH algorithm based on the EGM (1,1). The GPHBH (1,1) uses even grey model parameters, objective function evaluations, black hole algorithm structure, and tabu search memory for the event horizon condition. Therefore, the GPEA (1,1) can use the EGM (1,1) model to foresee the next generation. To evaluate the analysis of the GPHBH (1,1), the experimental study is conducted on eight benchmark traveling salesman problem (TSP) datasets. In the literature, a new discrete grey forecasting model named the DGM model and a group of optimized models have been proposed. DGM model and its optimized models can effectively be used to predict. Grey forecasting theory is applied to improve and suggest more new grey forecasting models (Xie & Liu, 2009).

The TSP is a combinatorial optimization problem that needs heuristic and metaheuristic algorithms to solve near optimal/or acceptable. Besides, TSP is a benchmark problem that is frequently used in comparative studies (Elloumi et al., 2014). The black hole algorithm and its hybrids are the common optimization algorithms that are frequently used in combinatorial problems. In this article, tabu search improves black holes during the event horizon by the size of tabu length. Moreover, black hole has a compact structure, and a smaller number of parameters than other algorithms. Consequently, it is a popular algorithm used in engineering, science, and technology.

The remaining part of the manuscript is organized as follows: In Section 2, the GPEA is explained. In Section 3, the GPHBH algorithm is discussed. The experimental results are presented in Section 4. Finally, in Section 5, the conclusion and recommendations are given.

2. The Grey Prediction Evolution Algorithm

The EGM (1,1) is the base prediction model that is used to obtain alternative heuristic solutions on the GPEA (Hu et al., 2020). The model collects the sequence of the raw data by objective function evaluation, and then uses a special operator, for instance, minimum selection of the accumulating generation on the discrete dataset in combinatorial optimization.

Data is transformed via the 1-AGO operator using the Eq. 1.

$$y^{(0)} = \left\{y^{(0)}(1), y^{(0)}(2), ..., y^{(0)}(n)\right\} \tag{1}$$

In the discrete optimization model, raw data is collected by the sequence of objective values in Eq. 2.

$$\mathrm{Obj}(x)^{(0)} = \left\{\mathrm{Obj}(x)^{(0)}(1), \mathrm{Obj}(x)^{(0)}(2), ..., \mathrm{Obj}(x)^{(0)}(n)\right\} \tag{2}$$

The sequence $y^{(1)}$ is regarded as intermediate information using the Eq. 3 and Eq. 4.

$$y^{(1)} = \left\{y^{(1)}(1), y^{(1)}(2), ..., y^{(1)}(n)\right\} \tag{3}$$

where

$$y^{(1)}(k) = \sum_{i=1}^{k} y^{(0)}(i), \qquad k = 1, 2, ..., n \tag{4}$$

In the discrete optimization model, intermediate information is the first-order accumulating generation operated by the minimum selection operator in Eq. 5.

$$y^{(1)}(k) = \min.\left(\mathrm{Obj}(x)^{(0)}(i)\right), \quad i = 1, 2, ..., k \wedge k = 1, 2, ..., n \tag{5}$$

Even Grey Model (1,1) is used by the following Eq. 6.

$$y^{(0)}(k) + \alpha \cdot z^{(1)}(k) = \beta \tag{6}$$

The EGM parameters are the grey coefficient α and the grey parameter β to be estimated, and $z^{(1)}(k)$ is the k^{th} element of the mean sequence $z^{(1)}$ in Eq. 7.

$$\begin{gathered} z^{(1)} = \left\{z^{(1)}(2), z^{(1)}(3), ..., z^{(1)}(n)\right\} \\ z^{(1)}(k) = 0.5 \cdot y^{(1)}(k) + 0.5 \cdot y^{(1)}(k-1), \qquad k = 2, 3, ..., n. \end{gathered} \tag{7}$$

The time response function is calculated and $\hat{y}^{(1)}(k)$ states the prediction sequence of the transformed data sequence $y^{(1)}$ in Eq. 8.

$$\hat{y}^{(1)}(k) = \left(y^{(0)}(1) - \frac{\beta}{\alpha}\right) \cdot e^{-\alpha \cdot (k-1)} + \frac{\beta}{\alpha}, \quad k = 1, 2, ..., n \tag{8}$$

The prediction sequence of the raw data is calculated and $\hat{y}^{(0)}(k)$ states the prediction sequence in Eq. 9.

$$\hat{y}^{(0)}(k) = \left(y^{(0)}(1) - \frac{\beta}{\alpha}\right) \cdot (1 - e^{\alpha}) \cdot e^{-\alpha \cdot (k-1)}, \quad k = 1, 2, ..., n \tag{9}$$

The $y^{(0)}(1)$ substitutes with $\mathrm{Obj}(x)^{(0)}(1)$ in the discrete optimization model in Eq. 8 and Eq. 9. Finally, a selection operator is implemented to obtain the candidate solutions in the next generation. In the next step, the GPEA combines with the hybrid black hole algorithm. Further, there exists a multi-variable grey prediction evolution algorithm for the metaheuristic in the literature (Xu et al., 2020).

3. The Grey Prediction Hybrid Black Hole Algorithm

In this work, the GPHBH is a popular metaheuristic that has been proposed for discrete optimization problems. The GPHBH mimics the behavior of stars turning around the black hole. The stars having the best fitness are replaced with the black hole, and this process continues until the best solution is found (Hatamlou, 2013). The tabu search improves the event horizon condition by a limited size of tabu memory. Some candidates in the population are replaced with the candidates in the tabu list (L), and the tabu list candidates are replaced with the current generation (Aladag et al., 2009).

The original updating position equation of each star turning around the black hole in the continuous problems, and global optimization is taken by using Eq. 10.

$$x^{i}_{\text{star}} = x^{i}_{\text{old}} + \text{rand} \cdot (x_{BH} - x_{\text{star}}) \tag{10}$$

The discrete updating position equation of each star turning around the black hole is taken by using Eq. 11.

$$\text{Obj}^{i}_{\text{now}} = \text{FObj}(1, \text{randperm}(N)) \tag{11}$$

In this work, more than one heuristic is applied once to the current generation, and the minimum value of the produced solutions is selected using Eq. 12. Those heuristics provide balanced exploitation and exploration in long iterations and CPU time in combinatorial optimization. However, those heuristics affect differently to each optimization problem.

$$\begin{aligned} \text{FNeigh}^{i,\text{iter}}_{\text{now}}(x) = \text{Apply the heuristics once} \\ \text{(swap,insert,reverse,swap-reverse)} \end{aligned} \tag{12}$$

The (2-opt, and 3-opt) heuristics are implemented to produce solutions and are evaluated with the objective function found via Eq. 9 in Eq. 13

$$\begin{aligned} \text{FNeigh}^{i,\text{iter}}_{\text{now}} < \text{Obj}^{i,\text{iter}}_{\text{now}} \\ \text{Obj}^{i,\text{iter}}_{\text{new}} = \text{FNeigh}^{i,\text{iter}}_{\text{now}} \end{aligned} \tag{13}$$

If the evaluated solution is better than the objective value found in Eq. 9, then the updated solution is the evaluated solution value. Otherwise, the new solution is updated as the past value.

In this work, the new generation is updated in four ways in event-horizon conditions for discrete problems in Eq. 14-18.

$$\frac{Q_{\text{Data}}}{|\text{Obj}_{BH} - \text{Obj}_i|} < \frac{f_{BH}}{\sum_{i=1}^{N} f_i} \tag{14}$$

$$\text{NewGeneration}_{\text{now}}^{i,t} = \text{TabuList}(i) \tag{15}$$

$$\text{TabuList}(i) = \text{Generation}_{\text{now}}^{i,t} \tag{16}$$

If the counter is over than the tabu size, the new generation is updated in Eq. 17.

$$\text{NewGeneration}_{\text{now}}^{i,t} = (1, \text{randperm}(N)) \tag{17}$$

If the event-horizon condition does not occur, the new generation is updated in Eq. 18

$$\text{NewGeneration}_{\text{now}}^{i,t} = \text{Generation}_{\text{now}}^{i,t} \tag{18}$$

3.1. Algorithm Description

Algorithm 1 shows the description of the pseudocode of the GPHBH algorithm using the grey prediction theory (Liu & Lin, 2011).

Algorithm 1. Pseudocode of the Grey Prediction Hybrid BH (GPHBH)

```
‣ Initialize GPHBH with nearest neighbor (NN) heuristic
‣ Initialize the trial population and generate a new population
‣ Initialize the Event horizon dataset parameter (Q), tabu size (L) and find
  the current best value
‣ While (Iteration <= MaxIteration)
   ‣ For k = 1 : (Population + Tabu size (L))
      ‣ α and β model parameters are estimated using Eqs. 6-7.
      ‣ y(1) (k) and y(0) (k) are computed using Eqs. 8-9.
      ‣ Update stars’ locations using Eq. 12.
      ‣ Apply an improvement heuristic (2-opt swap, 3-opt swap, and k-opt swap)
   ‣ End For k
   ‣ Decide the acceptance of the new generation using Eq. 13.
   ‣ Event-horizon condition occurs for discrete problems using Eq. 14-18.
   ‣ Rank Population individuals and find the best candidate in the total
     population
‣ End While
‣ State the final results (Final Statistics)
```

Similarly to the EGM (1,1), many grey prediction models (Liu et al., 2015) between the original grey model and the discrete grey model are discussed, and a new GPEA might be proposed.

In this work, two heuristics; swap and reverse intensify solutions in discrete space. However, insert and swap_reverse diversify solutions in discrete space. Each heuristic is applied sequentially in each step. As literature declares, the use of multiple heuristics is useful to improve solution quality and find competitive solutions in the discrete space (Halim & Ismail, 2019; Szeto et al., 2011). Then, applying two improvement heuristics; 2-opt swap, and 3-opt swap cause intensification in a scale of datasets ($n \leq 101$). However, 2-opt swap, and 3-opt swap cause diversification in other scale of datasets ($n > 101$). In the other side, nearest neighbor (NN) heuristic intensifies solutions in all medium-scale datasets.

4. Experimental Results

The eight datasets ranging from 38 to 150 cities were selected on a medium scale from the TSP Library in the application (Reinelt, 2013). In these problems, the x and y coordinates for the visited cities are given in the considered datasets. Euclidean distance was calculated using the x and y coordinates in the plane. The experiments were conducted 5 times independently with 64 GB RAM using Matlab. Table 1, Table 2, Table 3, Table 4, Table 5, Table 6 and Table 7 show the performance of the GPHBH algorithms such as GPHBH, GPHBH+2-opt, GPHBH+3-opt, GPHBH+NN+2-opt, and GPHBH+NN+3-opt.

All the proposed metaheuristics were conducted 5 times with 3000 iterations for each run. The population size is set to 100 for the hybrid algorithms. Tabu size is set to 20 for all datasets in the event horizon. However, the event horizon constant for the specific TSP dataset has been taken into consideration in computer system calculations. The parameters of algorithms are appropriate and adequate for those datasets and calculations as many works in the literature (Feng et al., 2019; Halim & Ismail, 2019; Hatamlou, 2018; Yildirim & Karci, 2018).

Table 1. Experimental solutions of grey prediction hybrid black hole algorithms for Eil51

Problem	Algorithm	Best Solution	Average Solution	Standard Deviation
Eil51	GPHBH	807.65	826.83	12.11
(426)	GPHBH+2-opt	536.52	576.38	23.81
	GPHBH+3-opt	512.39	531.34	17.17
	GPHBH+NN+2-opt	464.25	482.87	18.19
	GPHBH+NN+3-opt	457.17	482.42	15.9

Table 2. Experimental solutions of grey prediction hybrid black hole algorithms for Berlin52

Problem	Algorithm	Best Solution	Average Solution	Standard Deviation
Berlin52	GPHBH	13547.4	13769.6	136.42
(7542)	GPHBH+2-opt	9472.88	9862.29	328.12
	GPHBH+3-opt	9105.26	9468.44	403.6
	GPHBH+NN+2-opt	7892.31	8016.14	74.07
	GPHBH+NN+3-opt	7889.57	7940.85	46.97

Table 3. Experimental solutions of grey prediction hybrid black hole algorithms for St70

Problem	Algorithm	Best Solution	Average Solution	Standard Deviation
St70	GPHBH	1764	1787.55	23.41
(675)	GPHBH+2-opt	1081.35	1145.1	72.8
	GPHBH+3-opt	951.62	1020.78	47.08
	GPHBH+NN+2-opt	785.73	792.6	7.06
	GPHBH+NN+3-opt	762.26	778.77	15.39

Table 4. Experimental solutions of grey prediction hybrid black hole algorithms for Eil76

Problem	Algorithm	Best Solution	Average Solution	Standard Deviation
Eil76	GPHBH	1254.1	1286.31	21.47
(538)	GPHBH+2-opt	827.03	862.54	34.13
	GPHBH+3-opt	782.35	805.37	29.26
	GPHBH+NN+2-opt	634.64	634.84	0.13
	GPHBH+NN+3-opt	616.52	628.84	8.56

Table 5. Experimental solutions of grey prediction hybrid black hole algorithms for Eil101

Problem	Algorithm	Best Solution	Average Solution	Standard Deviation
Eil101	GPHBH	1738.92	1756	11.52
(629)	GPHBH+2-opt	1153.19	1168.5	13.94
	GPHBH+3-opt	985.32	1029.68	29.87
	GPHBH+NN+2-opt	744	761.01	10.58
	GPHBH+NN+3-opt	744.16	759.21	10.41

Table 6. Experimental solutions of grey prediction hybrid black hole algorithms for Bier127

Problem	Algorithm	Best Solution	Average Solution	Standard Deviation
Bier127	GPHBH	329503	332212	2915.43
(118282)	GPHBH+2-opt	215093	217992	2666.3
	GPHBH+3-opt	194387	201881	6691.12
	GPHBH+NN+2-opt	129942	130755	719.17
	GPHBH+NN+3-opt	129793	131358	2247.74

Table 7. Experimental solutions of grey prediction hybrid black hole algorithms for Kroa150

Problem	Algorithm	Best Solution	Average Solution	Standard Deviation
Kroa150	GPHBH	123595	125950	1704.36
(265249)	GPHBH+2-opt	72934.5	74050.4	942.04
	GPHBH+3-opt	63350.1	64881.5	2403.19
	GPHBH+NN+2-opt	30126.9	30699.6	376.74
	GPHBH+NN+3-opt	30820.2	31300.7	327.73

Table 1-7 shows the acceptable solutions of GPHBH algorithms. Eil101 states the number of cities for the dataset (EilN) in the TSPLIB. Though the best-known solutions were found after long calculations using exact methods, grey prediction hybrid algorithms converge to acceptable results (Eil51-Kroa150). The optimal/or acceptable solution states the total traveling costs of the best-known traveling path. The experimental analysis shows that GPHBH+NN+3-opt outperforms the solutions of GPHBH, GPHBH+2-opt, GPHBH+3-opt, and GPHBH+NN+2-opt algorithms.

Table 1-7 demonstrates that GPHBH with 2-opt, and 3-opt heuristics are quite better than the pure grey prediction hybrid algorithm, GPHBH. Moreover, the quality of the solutions using the nearest neighbor heuristic (NN) named GPHBH+NN+2-opt, and GPHBH+NN+3-opt is fairly well without using that heuristic. All the proposed GPHBH algorithms search for optimal/or acceptable solutions within reasonable CPU time when computational time is taken under consideration.

It is inferred that GPHBH algorithms can solve the TSP effectively with more hybrid structures and other improvement heuristics, such as k-nearest neighbor (k-NN), nearest insertion (NI), k-opt, cuckoo search, bat algorithm, and lion optimization.

It would be expected that GPHBH hybridizing with heuristic algorithms gives different acceptable solutions to each mathematical optimization problem, such as scheduling, routing, location, and inventory allocation.

4.1. Statistical Comparison

First of all, the best and average deviations of GPHBH algorithms on medium-scale TSP datasets are presented and interpreted in this section. Then experimental analysis of GPHBH+NN+3-opt algorithm is given by best, average solution and CPU time. At last, the statistical analysis of several algorithms is given and interpreted by mean and median as a centralization metric, and coefficient of variation as a dispersion metric. Besides, the algorithms are compared by CPU time as an efficiency criterion.

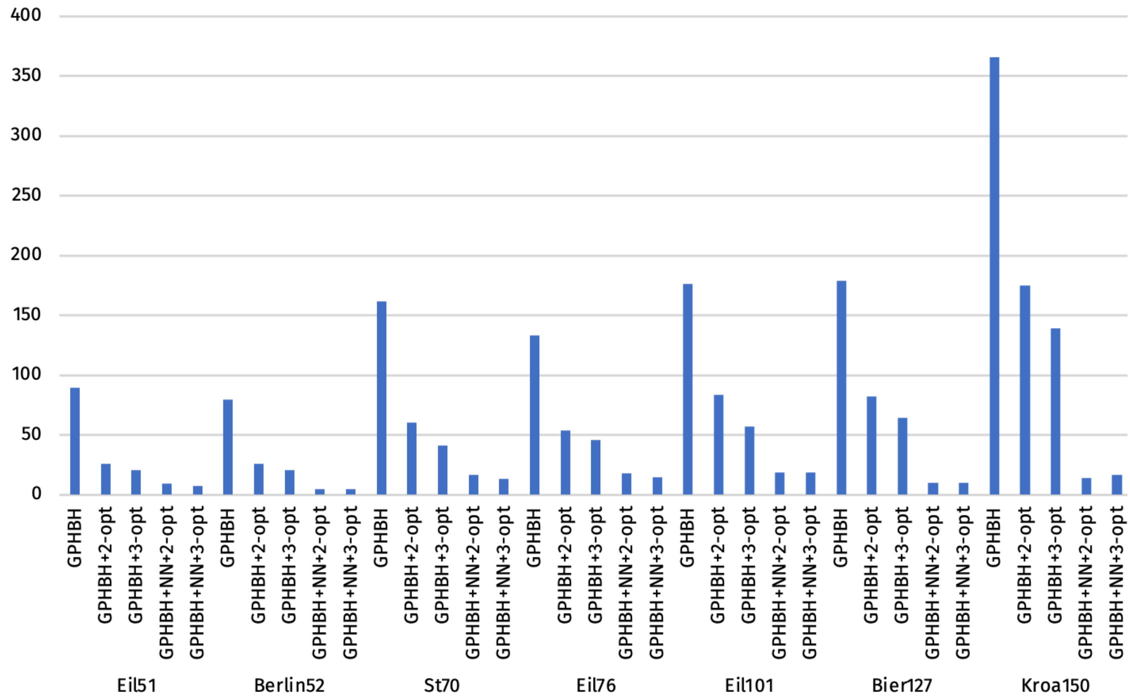

Figure 1. Best deviations of grey prediction hybrid black hole algorithms on medium-scale TSP datasets (Eil51-Kroa150)

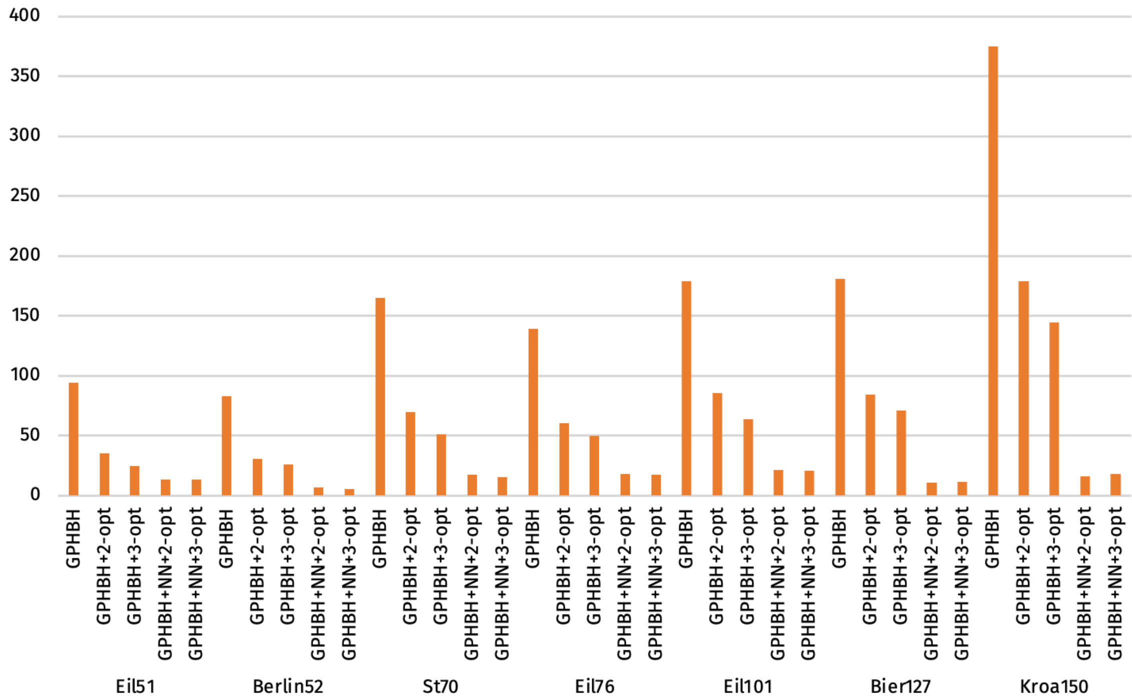

Figure 2. Average deviations of grey prediction hybrid black hole algorithms on medium-scale TSP datasets (Eil51-Kroa150)

Figure 1 and Figure 2 show the best and average deviations of GPHBH algorithms on medium-scale datasets. It is also concluded that slight differences are observed between best and average deviations when the hybrid metaheuristics (GPHBH, GPHBH+2-opt, GPHBH+3-opt), and the proposed hybrid algorithms (GPHBH+NN+2-opt, GPHBH+NN+3-opt) are investigated.

Table 8. Experimental solutions of GPHBH+NN+3-opt for all the medium-scale TSP datasets (Dj38-Kroa150)

Problem	Scale	Optimal Solution	Best Solution	Average Solution	CPU Time
Dj38	38	6656	6759.78	6881.79	23.78
Eil51	51	426	457.17	482.42	32.96
Berlin52	52	7542	7889.57	7940.85	40.68
St70	70	675	762.26	778.77	38.65
Eil76	76	538	616.52	628.84	42.17
Eil101	101	629	744.16	759.21	49.94
Bier127	127	118282	129793	131358	53.91
Kroa150	150	26524	30820.2	31300.7	65.16

Table 8 shows that the computational time performance of the GPHBH+NN+3-opt algorithm for the datasets is quite acceptable while the scale of the dataset increases. The traveling costs are relatively well when looking for new alternative solutions in the literature.

Table 9. Statistical comparison of all the metaheuristics on medium-scale TSP datasets (Dj38-Kroa150)

Dev./Time	GPCA	GPHBH	GPHBH + 2-opt	GPHBH + 3-opt	GPHBH + NN + 2-opt	GPHBH + NN + 3-opt
BDev.	154.9	157	64.6	48.8	11.8	10.9
Med. BDev.	146.9	147.2	57	43.2	17.2	13.8
AvDev.	161.5	162.2	71	54.5	14.1	13.2
Med. AvDev.	152.4	152	65	50.5	17.7	16.1
Coef. Var.	0.02	0.02	0.03	0.04	0.02	0.01
CPU Time	16.88	14.4	15.76	38.25	21.18	46.56

Table 9 shows that the GPHBH+NN+3-opt and GPHBH+NN+2-opt hybrid metaheuristics give acceptable routing deviations when compared to other metaheuristics. Coefficient of variation of those metaheuristics is quite small then they give stable and competitive solutions. Coefficient of variation of the GPHBH+3-opt metaheuristic is fairly higher than the GPHBH+2-opt, GPHBH, and GPCA metaheuristics, then it gives fluctuations and better solutions. The average results of GPCA, GPHBH, and GPHBH+2-opt algorithms are quite higher than the remaining algorithms, then they give low-quality solutions. The CPU time of the GPHBH+NN+2-opt algorithm is quite better and more reasonable than GPHBH+NN+3-opt and other algorithms to observe acceptable solutions.

5. Conclusion and Future Work

GPHBH algorithm which mimics the behavior of tabu search and black hole (BH) and its heuristic applications were introduced for medium-scale TSPs (Dj38-Kroa150). The GPEA uses the grey prediction theory, its simplest transformation operator (1-AGO), the transformed data sequence, the time response function, the prediction sequence of the raw data, and the prediction sequence of the transformed data sequence. The hybrid black hole algorithm uses tabu search in event horizon to keep a sample of solution in the tabu list and the tabu list is updated with the current generation. To produce new solutions, the minimum value of the heuristics such as swapping, insertion, 2-opt heuristic, and swap-reversing heuristic has been used. Besides, the k-opt improvement heuristic finds better solutions than the 3-opt heuristic, 3-opt heuristic finds higher quality solutions than the 2-opt heuristic. The GPHBHs with constructive heuristics such as k-NN, NN (Nearest Neighbor), NI (Nearest Insertion), CI (Cheapest Insertion), and RI (Random Insertion) would also find more acceptable solutions than the algorithms without using them.

In this article, the GPHBH algorithm initializes with the trial population and selects a random population. The generation of a new population iterates in each step, alpha and beta parameters of the even grey model are estimated. The algorithm uses the reproduction operator, named *egm11 reproduction*. Firstly, raw data is collected via objective function evaluation, and intermediate information is gathered by the minimum operator. Then, the prediction sequence of the transformed data sequence is calculated, and the prediction sequence of the raw data is collected. Finally, a selection operator is applied to maintain the most promising candidate solutions in the next generation.

Euclidian distance was calculated to consider traveling costs using x-y coordinates in the plane. All the statistics have been given as the best, average solution, standard deviation, and CPU time when the algorithm runs 5 times and 3000 iterations. The best and average solutions of GPHBH+NN+3-opt are preferable to the solutions of GPHBH+NN+2-opt, GPHBH+3-opt, and GPHBH+2-opt. The standard deviations of GPHBH+NN+3-opt are also stable. The proposed grey prediction hybrid black hole algorithm, GPHBH+NN+3-opt, solves the combinatorial problem in acceptable time (Ave. CPU time= 43.41 secs.). The second proposed hybrid algorithm, GPHBH+NN+2-opt, solves the combinatorial problem at a reasonable time (Ave. CPU time= 21.97 secs.). The GPHBH+3-opt and GPHBH+2-opt give not satisfactory results when compared to the GPHBH with constructive heuristics.

In this work, many alternative solutions using hybrid methods are proposed for solving corresponding combinatorial optimization problems. Those alternative solutions are useful for security, flexibility, design, assembly, production, and other fields of engineering, science, and technology. The decision-makers have to choose between alternative optimal results to take right positions against real-world problems. Thus, they allow sub-optimal, near-optimal, and/or acceptable solutions. This situation brings comfort and rapidness to producers and consumers.

In future works, the grey prediction hybrid algorithms using different methods such as exact methods, mathematical programming, and other metaheuristics would obtain many alternative solutions. Therefore, they bring innovations to the existing literature. The new grey prediction metaheuristics could be implemented in medium-large scale combinatorial benchmark datasets, using random datasets, literature datasets, and real case instances. Besides, the computational time can be evaluated with more grey prediction algorithms and other metaheuristics. Furthermore, a vehicle routing problem or a shift scheduling problem will be solved through GPHBH and its hybrids, to try to improve the performance of the proposed hybrid algorithms.

Declarations

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

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