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Application of a New Quantitative Approach to Stock Markets: Minimum Spanning Tree

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ABSTRACT

The systems involving interacting agents such as food networks, scientific citations, social networks, communication networks, the Internet, and the companies interacting in stock portfolios have long been studied by many researchers under the concept of complex systems. Such systems are expressed in terms of weighted networks. The dense connections and entwined relations amongst the agents play important roles for forecasting or risk analysis. In this study we present a novel approach to determine hierarchical structure of Industrial sector in the globally operating stock market network. By using the subdominant ultra-metric topology emerge from the minimum spanning tree of the stock market network; it becomes possible to extract the important properties of this complex system. Moreover, we use the dynamic time warping distance to determine the taxonomy and to measure similarity between time series of the operating Industrial sectors. It is found that United States, United Kingdom, Netherlands and Denmark are the most dominant stock exchanges in Industrials sector. We also find three hierarchical clusters and then topologically analyze the structure of considered clusters.

Keywords:

Dynamic Time Warping, Hierarchical Clustering, Minimum Spanning Tree, Stock Exchanges, Topology Evolution

Yeni Bir Nicel Yaklaşımının Hisse Senedi Piyasalarına Uygulanması: Minimum Geren Ağaç

ÖZET

Gıda ağları, bilimsel atıflar, sosyal ağlar, iletişim ağları, internet ve etkileşen hisse senedi portföyleri gibi birbiriyle etkileşen bileşenleri içeren sistemler, karmaşık sistemler kavramı altında pek çok araştırmacı tarafından çalışılmıştır. Bu tür sistemler ağırlıklı ağlar ile ifade edilir. Yoğun ağlar ve dolaşık ilişkiler, bileşenler üzerine tahmin yapma veya risk analizi için önemli role sahiptir. Bu çalışmada sanayi sektörü hisseleri üzerinden küresel borsaların hiyerarşik yapısını elde etmek için yeni bir metot sunuyoruz. Borsa ağını minimum geren ağaçtan çıkarılan alt baskın ultrametrik topolojiyi kullanarak, karmaşık sistemlerin önemli özelliklerini çıkarmak mümkündür. Ek olarak sınıflandırma yapmak ve ele alınan sanayi sektörlerine ait zaman serileri arasındaki benzerliği ölçebilmek için dinamik zaman bükmesi uzaklığını kullandık. Sonuçta Amerika Birleşik Devletleri, İngiltere, Hollanda ve Danimarka borsalarının sanayi sektörü hisse senetleri açısından incelenenler arasında en baskın borsalar oldukları bulunmuştur. Ayrıca hiyerarşik üç küme bulduk ve bu kümelerin yapısını topolojik olarak inceledik.

Anahtar Kelimeler:

Dinamik Zaman Bükmesi, Hiyerarşik Kümeleme, Minimum Geren Ağaç, Borsalar, Topolojik Evrilme



1. Introduction

The stock market is a regulated and organized financial market where investors buy and sell shares of publicly traded companies. There are several relations and interactions between the market structures. The problem is whether there is a suitable scientific representation of the relevancy between stock markets that would allow one to have helpful information about the nature of the market. Recently, graph-theoretical representation and analysis of the financial markets has been studied by many authors. In 1999, Mantegna [1] suggested mapping of the stocks into the graph nodes. By using the Pearson's correlations between the particular pairs of stock returns, the relations between stocks are projected into the graph edges. Under the inspiration of Mantegna's paper, the studies [2-6] are prepared on various branches of financial markets by using minimum spanning tree (MST). The concept of the MST could be considered as a minimal graph that covers all nodes without loops. Moreover the MST allows us to visualize complex and sophisticated relations by presenting in an uncomplicated manner.

Recently, differing from previous studies some authors focus on MST diagram employing dynamic time warping. In [7] authors analyzed the relations between foreign exchange currencies before, during, and after the US subprime mortgage crisis. Sectorial hierarchy in Turkish stock exchange market (Borsa Istanbul) has been investigated by using MST based on dynamic time warping [8]. In this manner here we consider dynamic time warping instead of classical correlation to weight the edges of MST. The MST is constructed to visualize the relations of the stock prices changes in Industrials sectors between different stock markets. Then hierarchical tree is constructed for clustering. Finally determined clusters were analyzed through numerical calculations. In section 2, basic information about the considered data, MST, and methods for analyzing the structure of the clusters is provided. In section 3, the results are presented as MST diagram, dendrogram of hierarchical tree and analyzing table for the structure of clusters. In section 4, concluding remarks are discussed.

2. Data and Methodology

Research is conducted on 29 stock exchanges covering time period beginning from September 20, 2006 till September 20, 2016. The daily stock change rates of Industrials sector shares are investigated. The studied stock exchanges which involve Industrial sectors and data intervals are listed in Table 1.

Country	Symbol	Stock Exchange	Time Period
Australia	AU	S&P/ASX 200	Sep 20,2006-Sep 20,2016
Belgium	BE	BEL 20	Sep 20,2006-Sep 20,2016
Brazil	BR	Bovespa	Sep 20,2006-Sep 20,2016
Canada	CA	S&P/TSX	Sep 20,2006-Sep 20,2016
Cote D'Ivoire	CI	BRVM 10	Mar 07,2014-Sep 20,2016
Denmark	DK	OMXC 20	Sep 20,2006-Sep 20,2016
Finland	FI	OMX Helsinki 25	Jan 10,2012-Sep 20,2016
France	FR	CAC 40	Sep 20,2006-Sep 20,2016
Germany	DE	DAX	Sep 20,2006-Sep 20,2016
Hong Kong	HK	Hang Seng	Oct 26, 2011--Sep 20,2016
Iceland	IS	ICEX	Sep 20,2006-Sep 20,2016
Italia	IT	Italy 40	Aug 16, 2011-Sep 20,2016
Kuwait	KW	Kuwait	Jul 09, 2010-Sep 20,2016
Malaysia	MY	FTSE Malaysia KLCI	Oct 27, 2011-Sep 20,2016
Mexico	MX	IPC	Sep 20,2006-Sep 20,2016
Namibia	NA	Namibia All Shares	Sep 20,2006-Sep 20,2016
Netherlands	NL	AEX	Sep 20,2006-Sep 20,2016
Portugal	PT	PSI 20	Sep 20,2006-Sep 20,2016
Saudi Arabia	SA	Tadawul All Share	Jan 07, 2007-Sep 20,2016
Sweden	SE	OMXS 30	Sep 20,2006-Sep 20,2016
Thailand	TH	SET	Aug 16, 2011-Sep 20,2016
Turkey	TR	BIST 100	Sep 20,2006-Sep 20,2016
UK	GB	FTSE 350	Aug 15, 2011-Sep 20,2016
USA	US	NASDAQ	Sep 20,2006-Sep 20,2016
Venezuela	VE	Bursatil	Nov 14, 2011-Sep 20,2016

Table 1. List of countries, symbols, related stock exchanges and operating time period

Classical correlation coefficients are widely used to determine the similarity distance between the nodes of graphs. However calculation of classical correlation coefficients (Spearman, Pearson etc.) needs synchronous data. Sometimes obtaining the same length of time series would be impossible. To analyze such multivariate data, dynamic time warping might be a useful tool. Dynamic time warping algorithm, uses the two temporal sequences of daily logarithmic return of the closure prices of each stock exchange during given time period. So by using DTW algorithm given by [6,8], the distance matrix D is computed to measure the similarity among the asynchronous time evolution of each pair of stock exchanges for Industrials sector. Then computed distance matrix is represented by MST in Figure 1.

It is not easy to find clusters in the MST of investigated stock exchanges. Therefore by using hierarchical clustering we build a dendrogram presented in Figure 2, depending on dynamic time warping. With respect to the dendrogram, 19 investigated stock exchanges are separated into 3 clusters. It is seen that other 6 stock exchanges are grouping as 2 couples and 2 stock exchanges that does not have enough similarity to form a cluster. For topology evolution of clusters, 3 clusters formed by 19 stock exchanges are considered which are containing more than 2 stock exchanges. Hence 6 stock exchanges are omitted due to not having enough similarity distance with at least 3 stock exchanges. Once the clusters are obtained, the sub

MSTs for each cluster are also considered to analyze the topology evolution of each cluster.

To analyze the topology evolution of considered stock market clusters, here we state the evaluation criteria given by [7]. First, the mean similarity measure (MSM) for the $n \times n$ similarity matrix D , is defined as

$$L_{MSM} = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^n D_{ij}, \quad (1)$$

where n is the number of stock exchanges in the investigated cluster. Then *the normalized tree length (NTL)* is defined by [10] as,

$$L_{NTL} = \frac{1}{(n-1)} \sum_{D_{ij} \in \Omega} D_{ij}, \quad (2)$$

where Ω is the set of edges, and $n - 1$ denotes the number of edges in the considered MST. Finally to quantify the average minimal route between pairs of nodes, we give the definition of *the characteristic path length (CPL)* as,

$$L_{CPL} = \frac{1}{n(n-1)} \sum_{i,j:i \neq j} l_{ij}, \quad (3)$$

where l_{ij} is the number of edges in the shortest path between nodes i and j [11]. By finding CPL we may understand how compact the structure of investigated network. After all, values L_{MSM} , L_{NTL} and L_{CPL} for the determined 3 clusters are presented in Table 2. All computations and plotted graphs are obtained with the help of software Mathematica 11.

3. Results

In this section we first analyze the similarity degree of the stock exchanges focusing on Industrials sectors of 25 countries around the world is represented by the MST approach. Obtained MST diagram is demonstrated in Figure 1. Further dendrogram is obtained by using hierarchical clustering by dynamic time warping. Plotted dendrogram is presented in Figure 2. Finally the topology evolution of clusters is investigated and the results are presented in Table 2.

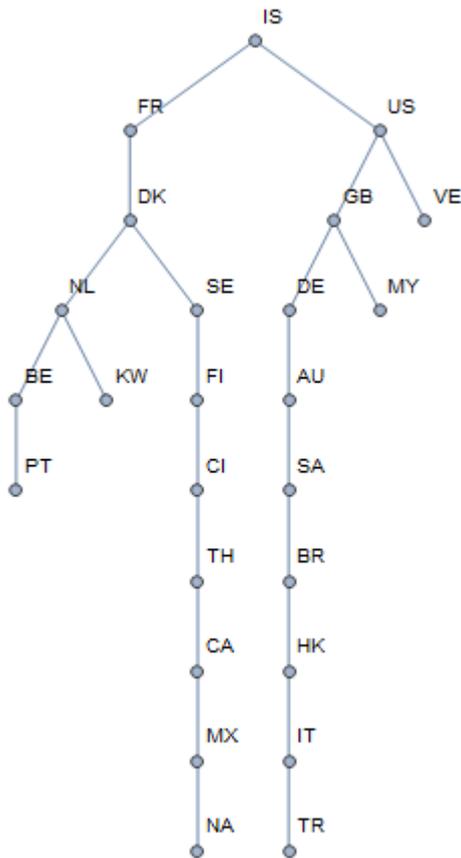


Figure 1. The MST diagram of stock exchanges focusing on Industrials sectors

Industrials sector stock shares of GB, US, NL and DK are in the central position of MST since they all have three edges. According to the MST diagram, GB stock prices are mainly affecting DE, US and MY stock prices. Similarly US is affecting GB, VE and IS. Another central position is DK since it directly affects FR, NL and SE. As the last center node of interacting stock markets, NL is similar with DK, BE and KW. These central position nodes (stock exchanges) have dominant hierarchy along the other nodes.

By investigating the hierarchical clustering depending on dynamic time warping, we reach the dendrogram presented in Figure 2. According to Figure 2, the 3 obvious clusters of stock exchanges are obtained. It is seen that DE, GB, US and MY clustering together. The second obvious cluster is BE, NL, DK, FR, IS, FI, SE and PT. The third cluster is consisting of CA, MX, CI, TH and NA. Moreover the couples AU, SA and BR, HK are forming clusters of 2 nodes. However these clusters are omitted for topology evolution analysis due to not having enough number of nodes. VE is near to the first cluster and so is KW to the second; however they are also not having the desired similarity with nearby clusters. The other exchanges IT and TR are neither able to form a cluster nor have enough similarity with other clusters.

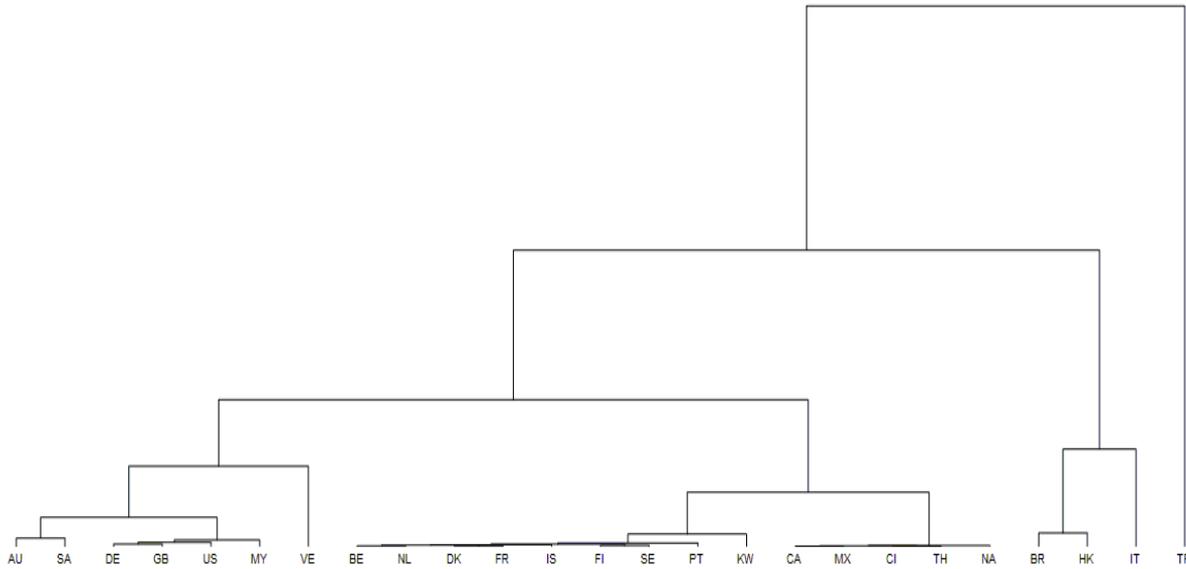


Figure 2. The hierarchical tree of stock exchanges focusing on Industrials sectors

The numerical comparison of MST properties of the considered 3 clusters is calculated in Table 2. Three network property measures namely, mean similarity measure, normalized tree length and characteristic path length are used to analyze the topology evolution of determined clusters. The average distance of edges in MST of the clusters could be found by normalized tree length. It is seen that third cluster has the smallest value so nodes in the third cluster are nearer to each other by comparing to other clusters. Further, regarding the Table 2 the characteristic path lengths are investigated. The results indicate that first cluster is the tightest and second one is the loosest cluster.

	Cluster 1	Cluster 2	Cluster 3
Mean similarity measure	1.03479×10^6	0.340159×10^6	0.885942×10^5
Normalized tree length	758600	181007	58688.5
Characteristic path length	2.5	3.57143	3

Table 2. Mean similarity measure, normalized tree length, characteristic path length

4. Conclusion

This study aims mathematical and quantitative approach to social sciences. By this way it is possible to have more reliable comparison of complex systems than classical methods. The description of complex systems may be more efficient by graph representation. It is known that visualizing the data has a positive effect on readers. Application of MST diagrams in socio-economic networks ensures more comprehensible data. Here we consider the MST diagram regarding on stock prices interactions in Industrials sectors among focused stock exchanges. Previous studies on stock exchanges usually construct MST diagrams depending on correlation coefficients. Calculation of correlation coefficients needs synchronous data to analyze. However sometimes one cannot obtain the synchronous data so it would be impossible to make comparison between such multivariate data. In this study to overcome the restrictions of classical correlation, a MST is constructed based on dynamic time warping which enables us to analyze asynchronous data. According to

obtained MST, it is seen that GB, US, NL and DK are located in the center of studied stock exchanges. It can be said that other studied 20 nodes (stock exchanges) are directly or indirectly connected to these 4 central nodes. Then hierarchical tree based on dynamic time warping is demonstrated through a dendrogram. Further, regarding to dendrogram hierarchical clusters were formed. Then the structures of clusters were analyzed by numerical calculations. With respect to the results it is found that US, DE, GB and MY indices are close and forming a cluster. We realized that second cluster grouped similarly with the geographical regions of corresponding countries in the world. Following stock exchanges are included in the second cluster BE, NL, DK, FR, IS, FI, SE and PT. Also CA, MX, CI, TH and NA are forming another cluster. Finally it is found that the nodes of first cluster are the closest to each other, and the third cluster is the tightest cluster.

References

- [1]. Mantegna, R.N. "Hierarchical structure in financial markets" *Eur. Phys. J. B* (1999) 11(1) pp: 193-197. doi:10.1007/s100510050929
- [2]. G. Bonanno, G. Caldarelli, F. Lillo, R.N. Mantegna "Topology of correlation-based minimal spanning trees in real and model markets", *Phys. Rev. E*, 68 (4) (2003), p. 046130 doi:10.1103/PhysRevE.68.046130
- [3]. Micciche, S., Bonanno, G., Lillo, F., Mantegna, R. N. "Degree stability of a minimum spanning tree of price return and volatility". *Physica A*. 2003, vol.324, pp.66–73. doi: 10.1016/S0378-4371(03)00002-5
- [4]. J.-P. Onnela, A. Chakraborti, K. Kaski, J. Kertész,, and A. Kanto, "Asset Trees and Asset Graphs in Financial Markets", *Physica Scripta*. (2003) Vol. T106, pp. 48–54. doi: 10.1238/Physica.Topical.106a00048
- [5]. J.-P. Onnela, A. Chakraborti, K. Kaski, and J. Kertész,, "Dynamic asset trees and portfolio analysis" *Eur. Phys. J. B* (2002) Vol. 30, pp. 285-288. doi: 10.1140/epjb/e2002-00380-9
- [6]. Reřovský, M., Horváth, D., Gazda, V., & Siničáková, M., "Minimum Spanning Tree Application in the Currency" Market. *Biatic*, 21(7) (2013), 21-23.
- [7]. Wang, G.J., Xie, C., Han, F., Sun, B., Similarity measure and topology evolution of foreign exchange markets using dynamic time warping method: Evidence from minimal spanning tree, *Physica A: Statistical Mechanics and its Applications*, Vol. 391 (2012), pp. 4136-4146.
- [8]. Hatipoglu V.F., "Sectorial Hierarchy in Borsa İstanbul", 2nd International Annual Meeting of Sosyoekonomi Society, October 28-29, 2016, Amsterdam
- [9]. D.J. Berndt, J. Clifford, Using dynamic time warping to find patterns in time series, *Workshop on Knowledge Discovery in Database*, 1994, pp. 359–370.
- [10]. W. Jang, J. Lee, W. Chang, Currency crises and the evolution of foreign exchange market: evidence from minimum spanning tree, *Physica A* 390 (2011), pp.707–718.
- [11]. J. Kwapień, S. Gworek, S. Drożdź, Structure and evolution of the foreign exchange networks, *Acta Physica Polonica B* 40 (2009), pp.175–194.