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Using Social Choice Function for Multi Criteria Decision Making Problems

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Abstract

This study comparatively examines the performance of social choice functions such as Borda, Copeland, Dodgson, and Kemeny in aggregating rankings in Multi-Criteria Decision Making (MCDM) problems. The analyses, conducted using a total of 500,000 datasets, observed that the aggregation results of different social choice functions were generally similar. Although the Borda and Copeland techniques are advantageous in terms of ease of application, they were found to be insufficient in obtaining a complete ranking, especially as the number of alternatives increases. This situation is also valid for the Dodgson and Kemeny techniques. The findings of the study indicate that these techniques provide consensus in the aggregation of rankings but fail to achieve a complete ranking. In 78% of the ranking aggregations using the techniques considered, a complete ranking could not be obtained. Additionally, it was determined that the average rate of achieving a complete ranking was higher in datasets with an even number of rankings compared to those with an odd number of rankings, specifically for the Copeland and Dodgson techniques. This study evaluates the effectiveness of social choice functions in aggregating MCDM problems and provides significant insights for future research.

Keywords Social Choice Function, Aggregation Techniques, Borda, Copeland, Kemeny, Dodgson, MCDM

Jel Codes C61, C72, G11

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1. Introduction

Social choice is as old as human history. Individuals can use their choices as they wish. Individuals who are part of a group, on the other hand, have to come to a common decision with other members on matters concerning the group. Collective decisions can be between two people or between millions of individuals, such as in presidential elections. Bringing together individual choices to a common ground and to a state in which collective decisions can be made requires certain predefined rules. The social choice theory identifies, analyzes, and evaluates the rules used for collective decisions (Boehmer & Schaar, 2023; Li et al., 2017).

The modern sense can be traced to the 18th century works of Borda and Condorcet. Borda count is a technique in which choices are ranked based on a points system which leads to the selection of the choice with the highest score. Condorcet developed a majority rule system based on a pairwise comparison of choices and discovered the situation where no choice could be given, which is called the voting paradox. In the event of a voting paradox, no elections can be held. Various approaches have been developed by Dodgson (1876), Nanson (1883), Copeland (1951) among others to solve the voting paradox. The development of the Social Choice Theory in the modern sense began with Black (1948), Arrow (1951), May (1952), and Sen (1970). The question of how to bring together the different and competing preferences of a group forms the basis of social choice theory.

However, Arrow (1951), in his doctoral thesis titled "Social Choice and Individual Values," which systematized social choice theory, stated that social choice functions contain some inconsistencies under certain fair criteria and that a perfect social choice function cannot exist. He proved that it is impossible for a social choice function to simultaneously satisfy the following conditions (Penn, 2015). The actions of the theorem, known in the literature as Arrow's impossibility theorem, are briefly outlined below (Arrow, 1951).

N is a finite set of individuals, where $n \geq 2$ who want to reach a collective decision; X is a finite set consisting of at least three alternatives from which each individual makes their preference orders, and i^{th} is the display of the choice orderings. where; including $\succcurlyeq_i$ the expression $x \succcurlyeq_i y$ shows that the i^{th} individual or a voter prefers alternative x at least as much as y. In this case, the $\succcurlyeq$ symbol indicates a weak preference, the $\succ$ symbol shows a strong preference and $\approx$ represents indifference. It follows therefore that expression $x \succ_i y$ shows that the individual i prefers alternative x more strongly than alternative y while expression $x \approx_i y$ shows that the individual i is indifferent to alternatives x and y. $f(R_i)$ shows the preference ordering of all individuals (Sen, 1970).

Unrestricted domain: The preferences of all individuals need to be taken into account. For n individuals evaluating the set of alternatives x, the set f must contain all the possible orders. In short, set f should be $f(R_1, R_2, ..., R_n)$.

Weak Pareto efficiency: If for any $x, y \in X$, the preference of all n individuals is $x \succ_i y$ then $x \succ y$. In short, any preference chosen unanimously must win.

Independence of irrelevant alternatives: In a case where $f(R_1, R_2, ..., R_n)$ represent an order set in which n individuals evaluate x, y, z alternatives and $f(R_1^*, R_2^*, ..., R_n^*)$ represents a preference profile in which the same n individuals evaluate x, z alternatives, if $x \succ_i y \succ_i z$ then $x \succ_i z$. In short, in an election where x, y, z candidates are evaluated, the withdrawal of any can didate from the election should not affect the election results.

Non-dictatorship: The preferences of R_d individual in the $f(R_1, R_2, ..., R_d, ..., R_n)$ preference set should not dominate the preferences of n individuals.

This theorem imposes significant limitations on the design of social choice functions and highlights the difficulties of an ideal collective decision-making mechanism. However, social choice functions are actively used in many fields, especially in MCDM. The reasons and benefits of using social choice functions in MCDM are outlined below.

One of the main functions of multi-criteria decision-making (MCDM) techniques is to rank the considered alternatives based on the determined criteria. There are many MCDM techniques used to rank the alternatives which use different analytical processes. The different MCDM techniques follow different analytical processes, hence different rankings could be obtained for the same problem (Voogd, 1982; Moghimi & Taghizadeh Yazdi, 2017; Zanakis et al., 1998). Before the problems which the MCDM techniques are expected to solve are outlined, a series of questions arise that need to be handled first. The first and most important of these questions is the choice of the MCDM technique. This question usually has more than one answer. Once this question is answered, the next question arises over the advantages of the preferred MCDM technique over other non-selected techniques, or whether it is more effective than the others. One can avoid answering these questions by opting to use more than one MCDM technique suitable for the problem in question and to use aggregation techniques to reach a complete ranking (Wang et al., 2009).

The inputs of the aggregation techniques are the rankings obtained by the MCDM techniques and they give, as their output, a consensus/integrated ranking. Obtaining a complete ranking from rankings obtained by the individual MCDM techniques using an aggregation technique is called the integrated MCDM approach. It has been determined in the literature review that the most commonly used social choice functions in the field of MCDM are the Borda and Copeland techniques, and Table 1 supports this view.

Table 1. Studies that have used social choice functions in MCDM

No	References	Num. of MCDM Technique	Num. of Alternatives	Aggregation Techniques Used
1	Ustinovichius et al. (2007)	3	4	Borda, Copeland
2	Honarmande Azimi et al. (2014)	5	12	Borda*, Copeland*
3	Banihabib et al. (2016)	3	9	Borda*, Copeland
4	Azadfallah (2016)	4	5	Borda*, Copeland*
5	Tuş Işık & Aytaç Adalı (2016)	3	6	Borda, Copeland
6	Moghimi & Taghizadeh Yazdi (2017)	3	22	Borda*, Copeland*
7	Mostafaeipour & Jooyandeh (2017)	5	25	Borda*, Copeland
8	Zavadskas et al. (2017)	4	21	Borda*, Copeland*
9	Çakır & Özdemir (2018)	3	11	Copeland
10	Ömürbek & Akçakaya (2018)	4	22	Borda*
11	Supçiller & Deligöz (2018)	8	5	Borda, Copeland

No	References	Num. of MCDM Technique	Num. of Alternatives	Aggregation Techniques Used
12	Gök Kısa & Perçin (2020)	4	29	Borda*
13	Kiani et al. (2019)	3	9	Borda*, Copeland*
14	Barak & Mokfi (2019)	3	6	Borda
15	Dortaj et al. (2020)	3	10	Borda*, Copeland*
16	Tavana et al. (2020)	3	18	Copeland
17	Donyaii et al. (2020)	6	4	Borda, Copeland
18	Şahin (2021)	7	6	Borda, Copeland
19	Yakut (2020)	2	29	Copeland*
20	Aytekin & Orakçı (2020)	6	4	Borda, Copeland, Nanson, Average, Cook and Seiford, Kemeny
21	Firouzi et al. (2021)	3	11	Borda, Copeland
22	Ecer (2021)	6	10	Borda, Copeland
23	Almutairi et al. (2021)	4	15	Borda*, Copeland*
24	Albulescu et al. (2022)	3	6	Borda, Copeland

* tied ranking

Table 1 shows that Borda and Copeland techniques were used in the majority of MCDM studies in which a single ordering was obtained through aggregation. In 12 of the studies a complete ranking could not be obtained with only one aggregation technique. It was also found that 11 of the studies in which a complete ranking of the results of the MCDM could not be obtained had more than 9 alternatives. Complete rankings were obtained from the aggregation of the MCDM results of 11 of the studies. 7 of the studies in which a complete ranking was obtained were found to have less than 7 alternatives. It can be concluded from the two cases above that when the aggregation is performed using Borda and Copeland techniques, the possibility of getting a collective ranking decrease with the increase in the number of alternatives.

As seen in Table 1, complete rankings are generally not obtained in MCDM problems where social choice functions are used. It can be said that in studies where complete rankings were not obtained, the inability to achieve a complete ranking in the aggregation of rankings relatively increases as the number of MCDM techniques and the number of alternatives used in the solution increase. However, determining these reasons in more detail constitutes the main objective of the study. To determine in which situations complete rankings cannot be achieved in the aggregation of MCDM results by social choice functions, a comparative analysis of the results of social choice functions will be conducted. In this analysis, alongside the commonly used Borda and Copeland techniques in the aggregation of MCDM results, the Dodgson and Kemeny techniques, which are more useful in problems with relatively fewer alternatives, will also be used. As a result of these analyses, the similarities in the ranking aggregation results of the social choice functions used will also be determined. The similarities of the aggregation results will be evaluated using the Kendall's coefficient of concordance test (W Test). Below, brief information on the steps of the W test and how it is used is provided.

Kendall's coefficient of concordance (W test), a non-parametric statistic is the equivalent of the ANOVA test and is used to determine the concordance between the assessments of more than three decision-makers who are presented with a certain preference profile to rank (Kendall & Smith, 1939). The hypothesis tests used for the W test are given below.

H_0 : There is no agreement between the rankings

H_1 : There is a complete agreement between the rankings.

If $p < 0.05$, then there is sufficient evidence to reject hypothesis H_0 and there is an agreement between the rankings. The fit scale of the W test is given in Table 2 (Duleba & Moslem, 2018). When the value of the W test is below 0.50, the p-value is usually greater than 0.05 and this usually leads to a significant W test result if there is a strong agreement.

Table 2. Kendall W concordance degree scale

W	Interpretation
0.0	No agreement
0.1	Weak agreement
0.3	Moderate agreement
0.6	Strong agreement
1.0	Perfect agreement

Table 3 outlines the steps and formulas that should be applied in the Kendall W test rankings in the presence or absence of complete ranking (Sidney, 1957).

Table 3. Steps for Kendall's Coefficient of Concordance

Parameter	description
s	Shows the deviations of the rank sums of the alternatives.
k	Number of rankings
N	Number of alternatives
W	Kendall W Coefficient of Concordance
T	It is the value that should be calculated when there is no exact consensus between the rankings.
	When there is a complete ranking: $s = \sum\left(R_j - \frac{\sum R_j}{N}\right)^2$, $W = \frac{s}{(\frac{1}{12})k^2(N^3-N)}$ If there is no complete ranking: $s = \sum\left(R_j - \frac{\sum R_j}{N}\right)^2$, $T = \sum\frac{(t^3-t)}{12}$, $W = \frac{s}{(\frac{1}{12})k^2(N^3-N)-k\sum T}$

2. Aggregation of Social Choices

Social choice functions are those in which individual preferences are evaluated together and which enable collective decisions. Social choice functions don't just stop at the selection of the winning candidate, they also indicate how the other candidates ranked against each other. Social choice functions have different properties, and this may lead to different results if different social choice functions are applied to the same rankings (Heckelman & Miller, 2015). In Table 4, voting rules, social choice functions, and some definitions are briefly explained. (The notations were explained in the previous section)

Table 4. Social election functions, rules and some definitions

Condorcet	The Condorcet winner is the most preferred candidate in pairwise comparisons against all other candidates. Where $x, y, z \in X$; $x \succ y$ and $x \succ z$ then x is the Condorcet winner.
Condorcet Paradox	A Condorcet paradox arises when no candidate is chosen or when these candidates cannot gain an advantage over each other. In the pairwise comparison of the candidates, it occurs when candidate x is preferred over candidate y, candidate y over candidate z, and candidate z over candidate x. The Condorcet paradox is said to occur where the preferences are cyclic $x \succ_i y \succ_i z \succ_i x$.
Condorcet Consistency	The Condorcet winner from all the voting methods or choice functions may not be named the winner. The technique that declares the Condorcet winner as the overall winner is considered a Condorcet Consistent technique. While Copeland, Kemeny, Nanson, and Baldwin are considered Condorcet consistent techniques, Borda and Dodgson are not Condorcet consistent techniques (Rossi et al., 2011; Gaertner, 2006). In other words, the Condorcet winner, who ranks first or is the most preferred candidate in pairwise comparisons also ranks first in Copeland, Kemeny, Nanson, and Baldwin techniques, whereas the order could change in Borda and Dodgson techniques.
Plurality	Where $x, y, z \in X$,$n = A$ and $x + y + z = A$. If $x > y$ and $x > z$, x is the most preferred candidate. Where 100 people take a collective decision vote on three candidates x, y, z as $x = 40$, $y = 32$ and $z = 28$, candidate x is the choice that wins with the plurality vote.
Majority	Where $x, y, z \in X$,$n = A$ and $x + y + z = A$. If $x > y + z$, x becomes the majority candidate. Where 100 people take a collective decision vote on three candidates the majority winner is the candidate who is preferred by at least $\frac{100}{2} + 1 = 51$ of the people.
Plurality with runoff	It is an iterative process that starts with the elimination of the least preferred candidate in the first round and one of the two last remaining candidates is declared the Condorcet winner.
Borda	In the aggregation of the rankings with the Borda rule, points are given to the alternatives according to their orders and they are ranked according to the sum of these points. In an aggregation with m alternatives, the most preferred alternative is given $m - 1$ points, the second most preferred alternative is given $m - 2$ and alternative ranked i is given $m - i$ points.
Dodgson	The Dodgson winning candidate is closest to the Condorcet winning candidate. Dodgson winner candidate is the candidate who needs the least preference change to be the Condorcet winner candidate ((Bartholdi et al., 1989). In the following preference profiles between four candidates A, B, C, D; $A > B > C > D$, $B > D > A > C$ and $C > D > A > B$, candidates A, C and D need at least two preference changes to become the Condorcet winner while alternative B needs at least one preference change. So, candidate B who is the closest to becoming the Condorcet winner becomes the Dodgson winner.
Copeland	Copeland winner is the candidate who wins the most in pairwise comparisons. Copeland points are determined for each alternative as follows; where $x, y \in X$ the Copeland score for alternative x is calculated as; $\sum(x \succ_i y) - \sum(y \succ_i x)$.
Nanson and Baldwin	The Nanson and Baldwin rules consist of the iterative process of the Borda function. In the Nanson rule, preferences below the average of Borda scores are eliminated and the process continues until only one preference remains. In the Baldwin rule, the process is done by eliminating the candidate with the smallest Borda score until only one choice remains (Rossi et al., 2011).
Kemeny	Kemeny (1959) proposed a technique that calculates the distance between the current rankings obtained in any given vote and all possible rankings. The closest ranking to the

possible rankings from the existing rankings is considered as the complete ranking. The distance between two rankings like R and R' is determined as; $d(R,R') = |(R-R') \cup (R'-R)|$. This can further be explored by defining two preferences j and k from the two rankings R and R' as $d(R,R') = \sum d(j,k)$. If there is a complete regarding alternatives and in the two rankings R and R', then $d(j,k) = 0$. However, if j is superior to k in R but k is superior to j in R', then $d(j,k) = 2$; if j is superior to k in R but j is indifferent to k in the R' ranking, that is, if the two preferences have the same order, $d(j,k) = 1$. (Bartholdi et al., 1989; Gaertner, 2006).

Applying different Multi-Criteria Decision Making (MCDM) techniques to the same problem leads to different results. Social choice functions are used to convert these different rankings into a single complete ranking. These functions help decision-makers combine the results obtained from different MCDM techniques, allowing for more consistent and reliable decisions, reducing conflicts between individual preferences, and assisting in forming a collective decision. The use of social choice functions contributes to obtaining more comprehensive and objective results in MCDM problems and adds significant value to studies in this field.

Situations that may arise from using social choice functions in the field of MCDM, the suitability of social choice functions for the MCDM field, and how these functions can be integrated into the decision-making process require in-depth analysis and research. The success of this integration depends on how well social choice functions reflect and address the complexity and dynamics of specific MCDM problems. For this purpose, a comparative analysis of social choice functions is conducted in Section 3.

3. Comparative Analysis of Social Choice Function in the Field of MCDM

In this section, the aim is to compare the results obtained by applying the Dodgson and Kemeny techniques, in addition to the Borda and Copeland techniques, which are frequently used in MCDM problems. For this purpose, 500,000 datasets containing 3, 4, 5, 6, 7, 10, 15, 20, 30, and 50 alternatives and 3, 4, 5, 6, 7 rankings were used. For each ranking, 100,000 datasets were randomly generated and produced through a script written in R. The R packages used include `irr`, `votesys`, `GGally`, and `MASS`. The dataset rankings obtained were aggregated using the Borda, Copeland, Dodgson, and Kemeny techniques. Kendall's W test was used to examine the relationship between the aggregation results of the techniques and the rate of obtaining a complete ranking. The purpose of these analyses and the potential benefits of the results obtained are outlined below.

Firstly, the aim is to compare the performance of widely used social choice functions such as Borda, Copeland, Dodgson, and Kemeny on datasets created with different numbers of alternatives and rankings. Another important objective is to evaluate the ability of social choice functions to provide consensus in aggregating rankings and to achieve a complete ranking. This can provide a significant advantage, especially when working with large and complex datasets.

The results of this study will demonstrate how social choice functions can be used more effectively in MCDM problems and will offer suggestions for improving the current methods in this field. Additionally, comparing the results obtained by using various MCDM techniques and social choice functions together will help us better understand the advantages and disadvantages of these techniques and determine which techniques are more suitable in which situations. This will contribute to decision-makers making more informed and accurate decisions. Moreover, this study on the applicability and effectiveness of social choice functions in the field of MCDM will provide important insights and directions for future research, significantly contributing to the literature and expanding the current knowledge in this field. In summary, this study will help us better understand the limitations of social choice functions.

The general representation of the R codes for the rankings used in the analysis and the compared techniques is given in Algorithm 1.

Algorithm 1. General Representation of The Codes

```
library(irr)
library(votesys)
library(GGally)
library(MASS)
  s1 <- c(…)
  s2 <- c(…)
  sn <- c(…)
  matrisA <- t(as.matrix(data.frame(s1,s2,…,sn)))
  matris_A <- create_vote(matrisA, xtype = 1 )
  B <- borda_method(matris_A)
  C <- cdc_copeland(matris_A)
  D <- cdc_dodgson(matris_A)
  Bs <- rank(B$other_info$count_min, ties.method = 'average')
  Cs <- rank(-C$other_info$copeland_score, ties.method = 'average')
  Ds <- rank(D$other_info$dodgson_quick, ties.method = 'average')
      }
    }
  }
  if(Alt<=8){
    K <- cdc_kemenyyoung(matris_A)
    Ks <- order(rank(K$other_info$win_link[1,], ties.method = 'average'))
    S <- data.frame(Bs,Cs,Ds,Ks)
  }else{
    S <- data.frame(Bs,Cs,Ds)
  }
    W <- c(kendall(S,correct = TRUE))
    W <- c(kendall(S,correct = FALSE))
```

Kendall W test was used to determine the similarities between the aggregation results. The analysis also included a test of whether the final rankings obtained from the aggregation of the rankings by the individual techniques had any tied situations.

The degree of agreement between the results obtained by Borda, Copeland, Dodgson and Kemeny aggregation techniques and the number of discordant datasets are given in Table 5.

Table 5. Kendall W test values

NoR	NoA	Min.	1st Q	Med.	Avg.	3rd Q	Max.	Discordant Sets
3	3	0.25	1	1	0.9502	1	1	526
	4	0.7353	0.9559	0.9888	0.9685	1	1	0
	5	0.25	0.95	0.9797	0.9665	0.9905	1	6
	6	0.65	0.9522	0.973	0.9664	0.9928	1	0
	7	0.689	0.9518	0.9715	0.965	0.9952	1	0
	10	0.8743	0.974	0.9833	0.9803	0.9898	1	0
	15	0.9021	0.9787	0.9848	0.9831	0.9895	0.9996	0
	20	0.9434	0.9819	0.9865	0.9854	0.9901	0.9982	0
	30	0.9594	0.9857	0.9888	0.9881	0.9912	0.9974	0
	50	0.9743	0.9893	0.9911	0.9908	0.9927	0.997	0
4	3	0.25	0.8167	0.9423	0.8732	1	1	649
	4	0.25	0.8716	0.9295	0.9036	0.9797	1	140
	5	0.25	0.8782	0.9295	0.912	0.9605	1	17
	6	0.3517	0.8913	0.9305	0.9187	0.9601	1	2
	7	0.4675	0.9002	0.9338	0.9239	0.9587	1	1
	10	0.7717	0.9592	0.9738	0.9687	0.9836	0.9986	0
	15	0.8685	0.9675	0.977	0.9742	0.984	0.9978	0
	20	0.8824	0.9728	0.9798	0.978	0.9852	0.9982	0
	30	0.9337	0.9788	0.9834	0.9823	0.9872	0.9965	0
	50	0.963	0.9839	0.9868	0.9862	0.9892	0.9953	0
5	3	0.25	0.95	1	0.9424	1	1	659
	4	0.4062	0.925	0.9808	0.9546	1	1	30
	5	0.25	0.9342	0.9633	0.9554	0.9905	1	8
	6	0.5761	0.9366	0.9653	0.9549	0.9802	1	0
	7	0.689	0.9395	0.963	0.9553	0.9787	1	0
	10	0.8102	0.9658	0.9774	0.9735	0.9857	1	0
	15	0.8985	0.974	0.9812	0.9793	0.9866	0.9988	0
	20	0.9387	0.9796	0.9844	0.9832	0.9883	0.9979	0
	30	0.9508	0.985	0.9881	0.9873	0.9905	0.9969	0
	50	0.9744	0.9898	0.9915	0.9911	0.9928	0.997	0
6	3	0.25	0.8167	0.9423	0.893	1	1	1094
	4	0.0625	0.891	0.9295	0.9154	0.9797	1	72
	5	0.3214	0.89	0.9359	0.921	0.9652	1	22
	6	0.49	0.9033	0.938	0.927	0.9638	1	2
	7	0.5739	0.9083	0.9398	0.9303	0.9627	1	0
	10	0.7829	0.9588	0.9733	0.9682	0.9833	0.9987	0
	15	0.8665	0.9685	0.9775	0.9749	0.9842	0.9988	0
	20	0.9082	0.9745	0.9809	0.9792	0.9858	0.9979	0
	30	0.9274	0.9812	0.9852	0.9842	0.9883	0.9962	0

NoR	NoA	Min.	1st Q	Med.	Avg.	3rd Q	Max.	Discordant Sets
	50	0.9688	0.987	0.9891	0.9887	0.9909	0.9965	0
7	3	0.25	0.95	1	0.9391	1	1	715
	4	0.4038	0.9118	0.9808	0.9481	1	1	47
	5	0.25	0.9261	0.9595	0.9477	0.9905	1	21
	6	0.5577	0.9299	0.9601	0.9491	0.9786	1	0
	7	0.6473	0.9326	0.9587	0.9497	0.9773	1	0
	10	0.7392	0.9617	0.9746	0.9704	0.9837	1	0
	15	0.787	0.9722	0.9797	0.9775	0.9854	0.9984	0
	20	0.8994	0.9781	0.9834	0.982	0.9874	0.9982	0
	30	0.9456	0.9843	0.9874	0.9867	0.9899	0.997	0
	50	0.975	0.9899	0.9915	0.9912	0.9928	0.9965	0

NoA: Number of Alternative, NoR: Number of Ranking

Table 5 shows that there is a lack of consensus in the aggregation results of the datasets consisting of 3, 4 and 5 alternatives while Table 4 shows a weak agreement. The number of discordant datasets is higher in the datasets with 4 and 6 rankings than in the datasets with 3,5 and 7 rankings.

The consensus between the aggregation results increases with the increase in the number of alternatives. There is also a positive correlation between the number of rankings and the aggregation results. We can therefore conclude that as the number of alternatives and the number of rankings increase, the consensus between the aggregation results increases.

Table 6 gives the number of datasets in which a complete ranking could not be obtained based on the aggregation techniques. The Kemeny technique is a distance-based aggregation technique hence the solution is obtained by determining the distance of the given rankings from an imaginary guidance vote. This implies that this technique does not suffer from the consensus problem since the guidance vote already has a complete ranking. However, since the distances of the rankings to the guidance vote may be the same, there may be more than one complete ranking with the Kemeny technique. Therefore, the results of the Kemeny technique could not be calculated in Table 6

Table 6. Number of datasets without complete rankings

Number of Ranking	Number of Alternative	Borda	Copeland	Dodgson
3	3	2233	526	1388
	4	4452	1710	3522
	5	6164	3306	5468
	6	7318	4827	7093
	7	8194	6505	8275
	10	9299	9248	9644
	15	9865	9988	9977
	20	9976	10000	9999
	30	9998	10000	10000
	50	10000	10000	10000

Number of Ranking	Number of Alternative	Borda	Copeland	Dodgson
4	3	3153	4190	6903
	4	4998	5596	8677
	5	6192	7030	9476
	6	7159	7929	9799
	7	7744	8489	9935
	10	8999	9550	9996
	15	9749	9929	10000
	20	9954	9986	10000
	30	9998	10000	10000
	50	10000	10000	10000
5	3	2469	659	1676
	4	4370	2139	3993
	5	5750	3934	5809
	6	6715	5729	7319
	7	7345	7362	8231
	10	8701	9688	9536
	15	9625	9998	9938
	20	9888	10000	10000
	30	10000	10000	10000
	50	10000	10000	10000
6	3	2583	3979	6012
	4	4299	5144	7910
	5	5429	6558	8933
	6	6320	7549	9446
	7	7044	8218	9726
	10	8483	9399	9978
	15	9476	9907	9998
	20	9829	9990	10000
	30	9982	10000	10000
	50	10000	10000	10000
6	3	2303	715	1744
	4	3842	2259	3956
	5	5077	4220	5689
	6	5988	6068	6981
	7	6683	7650	7969
	10	8223	9759	9374
	15	9281	9999	9887
	20	9758	10000	9984
	30	9963	10000	10000
	50	9999	10000	10000

As seen in Table 6, the number of datasets with complete rankings decreases as the number of alternatives increases, and a complete ranking was not found in almost all the datasets with more than 20 alternatives. The rate at which complete rankings are obtained using aggregation techniques depends on the number of alternatives. These rates of change are given in Table 7. The rates are grouped as datasets with few (3,4,5,6,7); medium (10,15,20) and many alternatives (30,50).

Table 7. Mean rates of failure to obtain a complete ranking based on the number of alternatives

Number of Alternative	Borda	Copeland	Dodgson
Few (3,4,5,6,7)	0,5353	0,4891	0,6637
Medium (10,15,20)	0,9407	0,9829	0,9887
Many (30,50)	0,9994	10000	10000

Table 7 shows that the probability of obtaining a complete ranking decreases with the increase in the number of alternatives. Although the probability of obtaining a complete ranking is relatively high in datasets with a few alternatives, it can be seen from the table that at least half of the datasets did not achieve a complete ranking. In datasets with a medium number of alternatives, the rate of not obtaining a complete ranking is at least 94%, while in datasets with a large number of alternatives, a complete ranking was not obtained in any of them. Furthermore, based on the aggregation technique, the average rate of not obtaining a complete ranking is 75% for the Borda technique, 74% for the Copeland technique, and 83% for the Dodgson technique. The overall average rate of not obtaining a complete ranking is approximately 78%. Therefore, it can be concluded that, especially in cases with a large number of alternatives, the Borda and Copeland techniques, which are widely used in MCDM problems, do not achieve a complete ranking.

Figure 1 shows that the average rate of obtaining a complete ranking decreases in datasets with an even number of rankings (4, 6) when using the Copeland and Dodgson techniques, while this average rate increases in datasets with an odd number of rankings (3, 5, 7). In other words, having an odd number of rankings increases the likelihood of obtaining a complete ranking, whereas having an even number of rankings decreases this likelihood. On the other hand, the Borda technique also shows a decrease in the average rate of obtaining a complete ranking as the number of rankings increases, but it does not exhibit a fluctuating pattern.

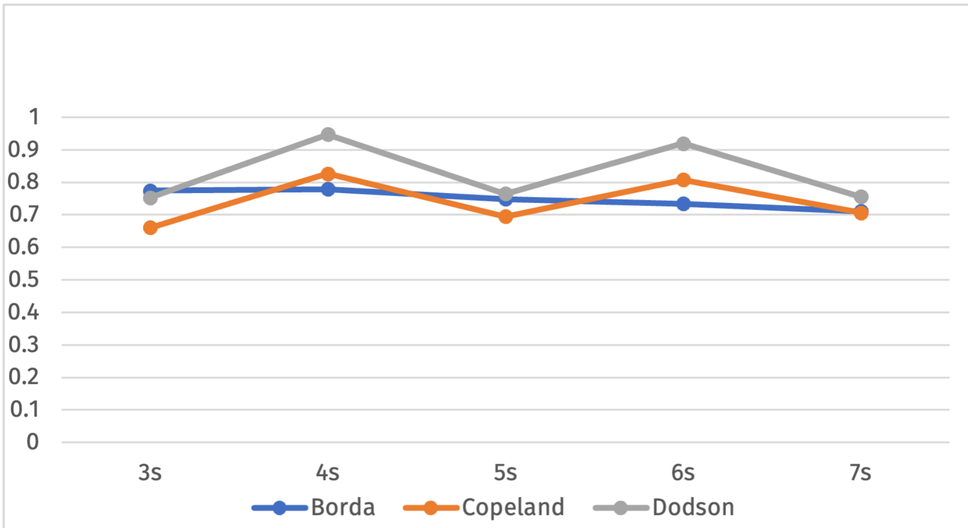

Figure 1. The Average of datasets for which consensus could not be obtained through aggregation techniques

The reason for the higher average rate of obtaining a complete ranking in datasets with an even number of rankings compared to those with an odd number of rankings when using the Dodgson technique can be explained by the fact that a candidate close to the Condorcet winner can be more easily determined in rankings with an odd number of rankings. The rankings of the datasets in Table 8 will help clarify this point more clearly.

Table 8.

	First Ranking					Second Ranking				
	s1	s2	s3	s4		s1	s2	s3	s4	s5
1	a	a	b	b	1	a	a	b	b	c
2	b	b	a	a	2	d	d	c	d	b
3	d	c	c	d	3	c	c	a	c	a
4	c	d	d	c	4	b	b	d	a	d

In the rankings in Table 8, the alternatives “a” and “b” are the Dodgson winners in the first ranking with an even number of rankings. In this case, a consensus cannot be achieved. The second ranking has an odd number of places, which increases the probability of breaking the tie, thereby increasing the probability that one alternative will be declared the Dodgson winner. In the second ranking, alternative “b” is the Dodgson winner as it is the closest candidate to the Condorcet winner. In short, the odds of becoming the Condorcet winner increase in favor of any candidate in rankings with an odd number of rankings.

In comparisons using the Copeland technique, since the Copeland winner is determined as the winner in the pairwise comparisons of the alternatives, the chances increase in favor of a candidate in rankings with an odd number of rankings. This also increases the probability of obtaining a complete ranking.

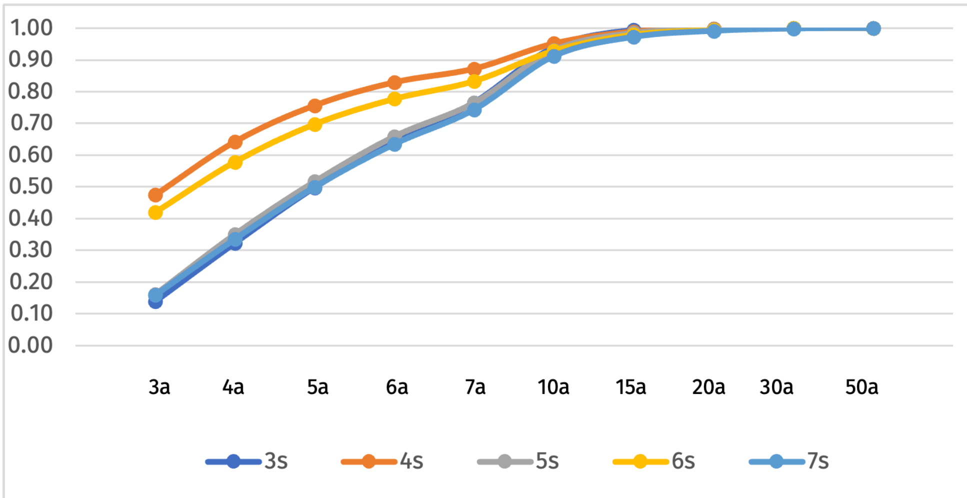

Figure 2. The average of failure to achieve consensus based on number of rankings.

In Figure 2, it can be seen that the average rates of not obtaining a complete ranking increase depending on the number of alternatives considered. Particularly in sets with more than 20 alternatives, it is common not to achieve a complete ranking. This situation is also clearly seen in Table 6. Additionally, according to the data in Figure 2, it has been observed that as the number of rankings in the samples increases, the average rates of not obtaining a complete ranking also increase. However, it can be said that the inability to obtain a complete ranking is more dependent on the number of alternatives.

4. Conclusion

According to Kendall's coefficient of concordance results, there is a high concordance in the aggregation of rankings among the techniques used in this study, including the frequently used Borda and Copeland techniques, as well as the Dodgson and Kemeny techniques in MCDM problems. However, it has also been determined that the average probability of not obtaining a complete ranking in the techniques included in the analysis is more than 78%. Although the Copeland technique has a higher probability than the Borda and Dodgson techniques in datasets with a small number of alternatives (3, 4, 5, 6, and 7), this rate was found to be approximately 49%. The Borda technique performed better than the Copeland and Dodgson techniques in terms of the probability of achieving a complete ranking in datasets with a moderate number of alternatives (10, 15, and 20), but even here, the probability of not obtaining a complete ranking was still 94%. In datasets with a large number of alternatives (30, 50), a complete ranking could not be obtained with the Copeland and Dodgson techniques, whereas the probability of not obtaining a complete ranking with the Borda technique in the same datasets was found to be 99.94%. Additionally, it was observed that the probability of not obtaining a complete ranking with the Copeland and Dodgson techniques in datasets with an even number of rankings was higher compared to datasets with an odd number of rankings. Given the high probabilities of failing to obtain a complete ranking, the suitability of the Borda, Copeland, Dodgson, and Kemeny techniques in the aggregation of MCDM problems becomes debatable.

The findings highlight the limitations and challenges in the literature regarding the use of social choice functions in the field of MCDM. The extensive datasets and analysis methods used in this study have comprehensively evaluated how social choice functions perform on datasets with different numbers of alternatives and rankings. The results underscore the limitations of these techniques in achieving complete rankings in MCDM problems and emphasize potential research areas for future studies.

Combining the rankings obtained with MCDM techniques is important because often more than one technique can be used in selecting the appropriate technique for the problem at hand. This study presents the results of using social choice functions in aggregating rankings in problems of different dimensions and shows that an increase in the number of alternatives has a negative impact on achieving a complete ranking. The results indicate that there is a need for more effective techniques for the aggregation of rankings in MCDM problems. These findings provide an important basis for understanding the limitations of current techniques and highlight the need for developing more effective methods in the field of MCDM.

Declarations

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

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